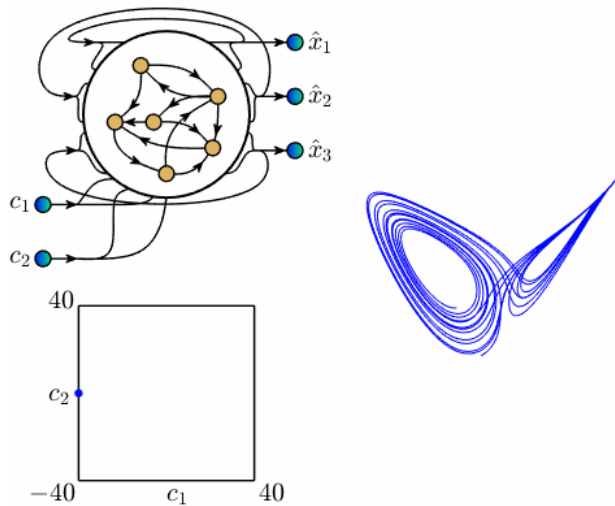


# Teaching recurrent neural networks to infer global temporal structure from local examples

Jason Z Kim, Zhixin Lu, Erfan Nozari, George J. Pappas, Danielle S. Bassett

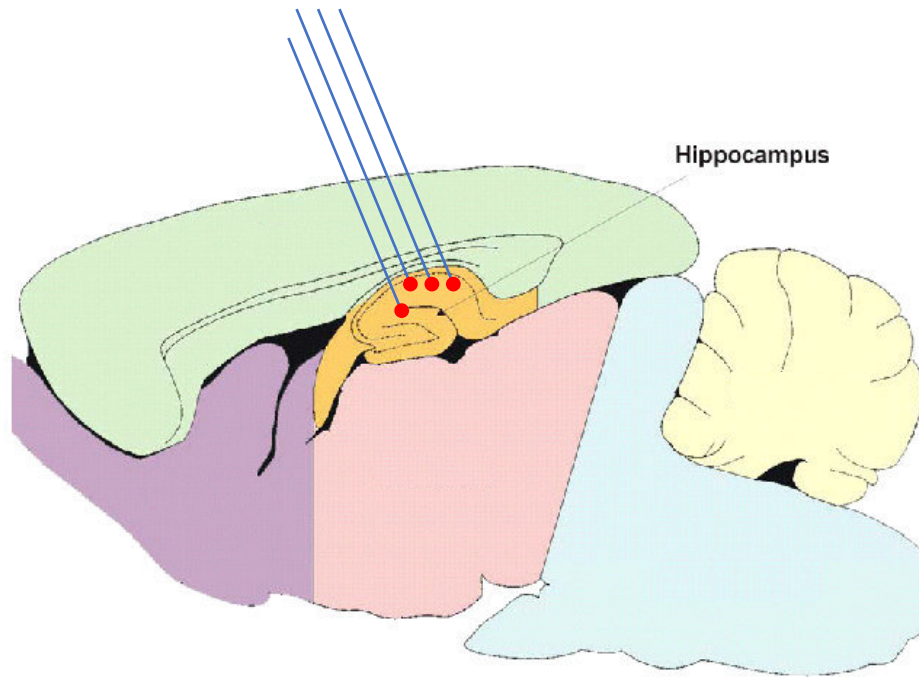


Complex  
Systems



# Neural systems sustain and manipulate memories

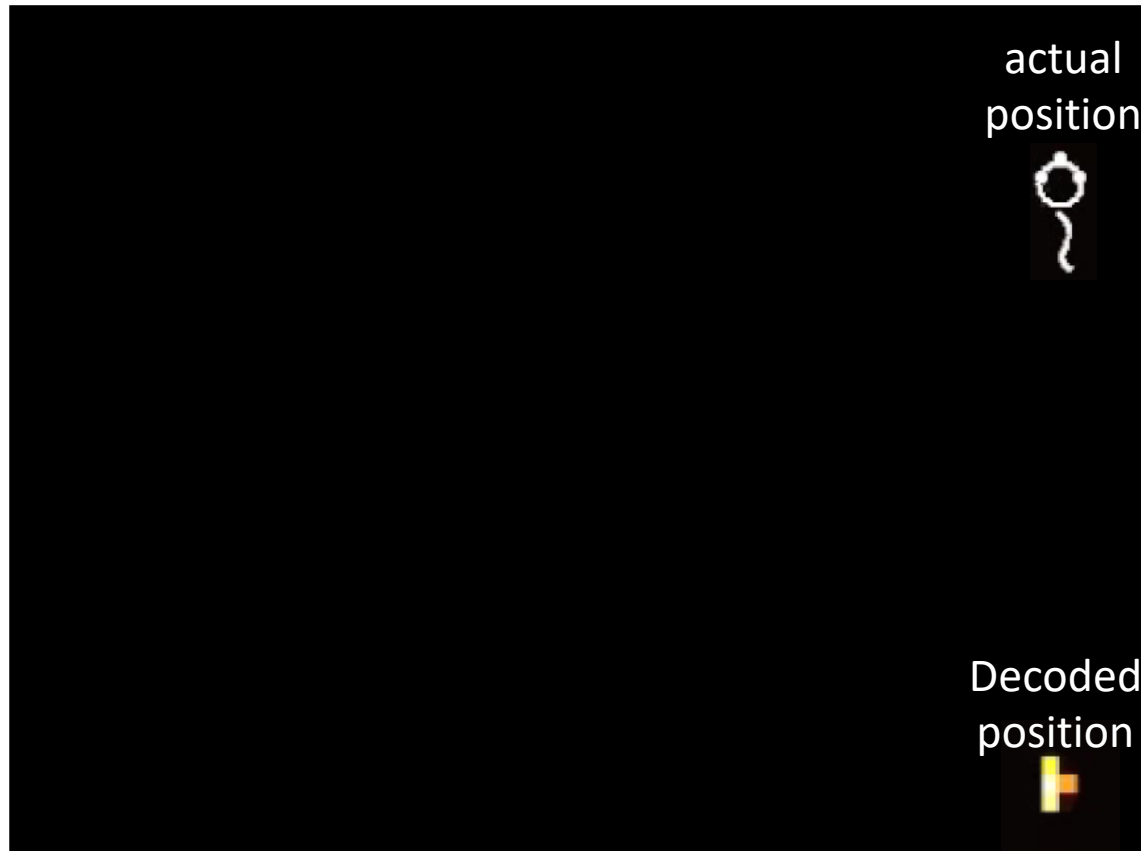
Spatial localization and forecasting





# Neural systems sustain and manipulate memories

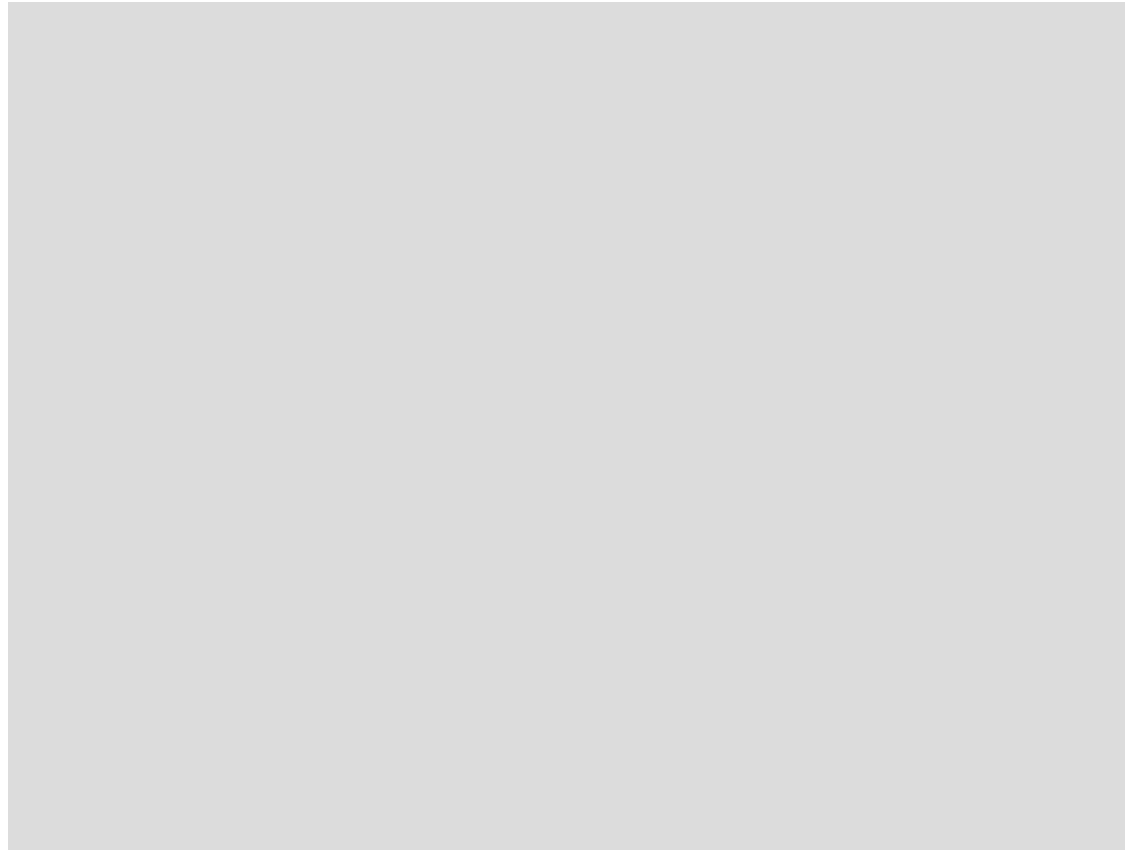
## Spatial localization and forecasting



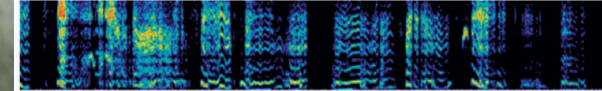
Pfeiffer, B. E., & Foster, D. J. (2013). Hippocampal place-cell sequences depict future paths to remembered goals. *Nature*, 497(7447), 74–79.  
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Spatial localization and forecasting



40 dph - subsong



Song

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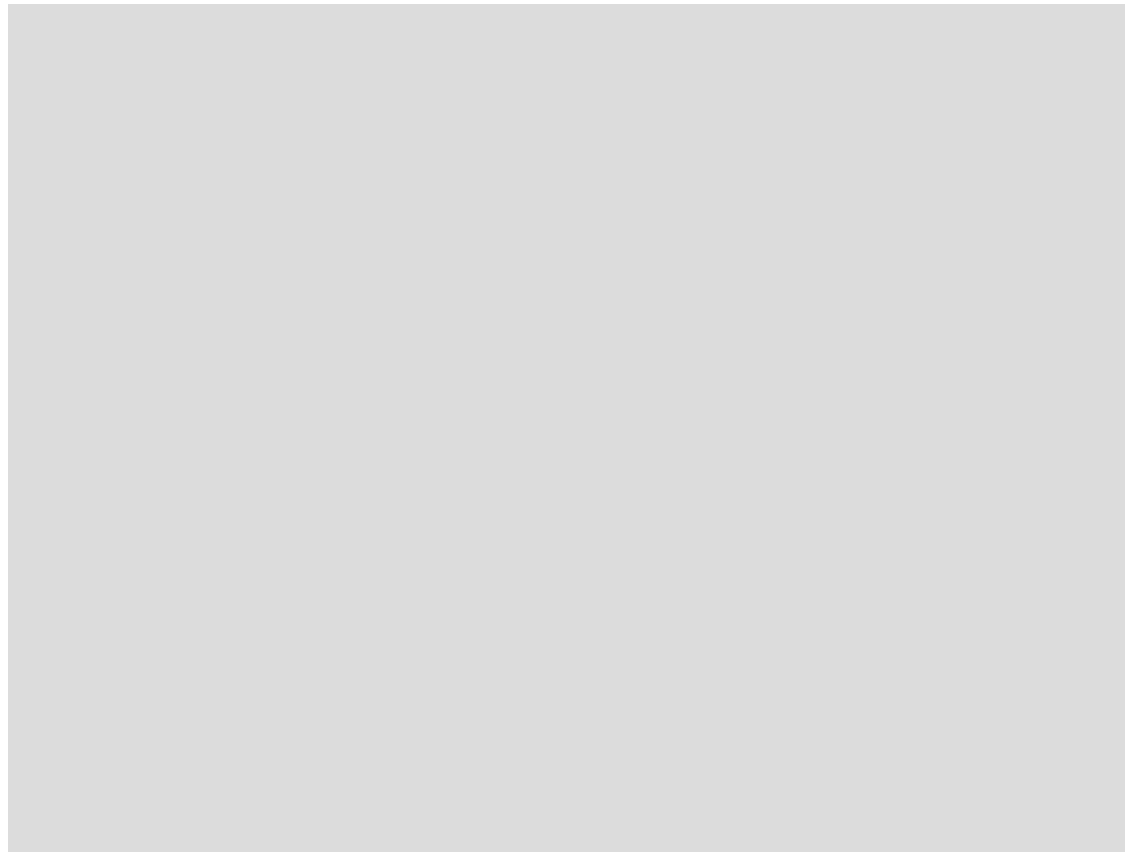
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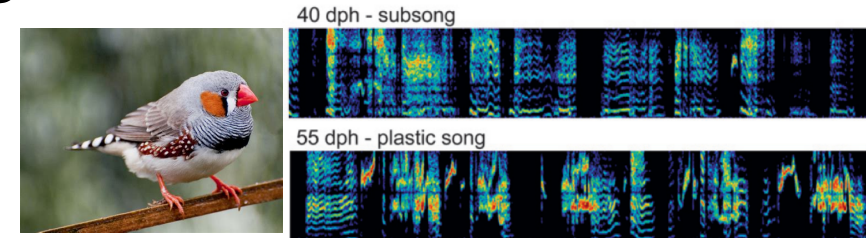
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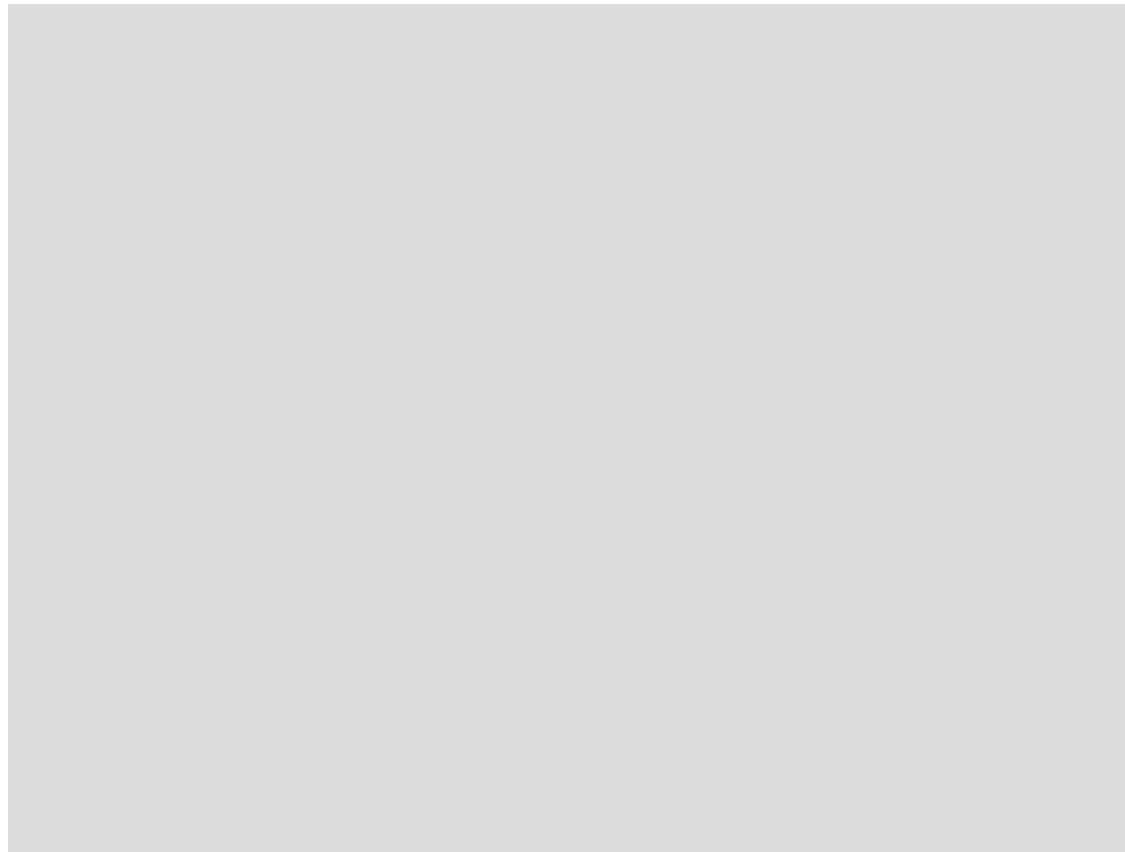
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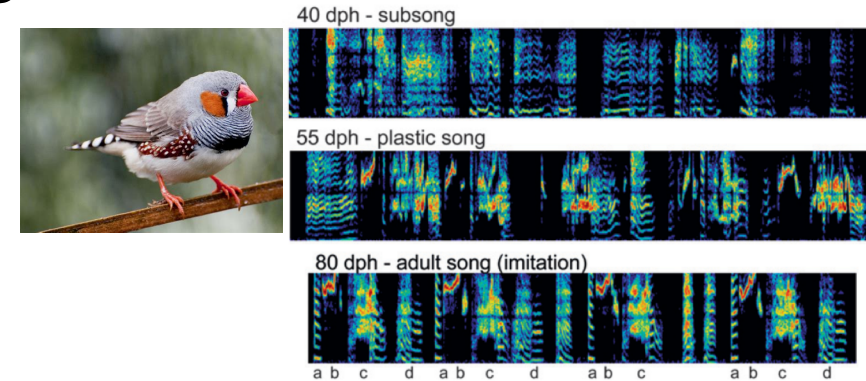
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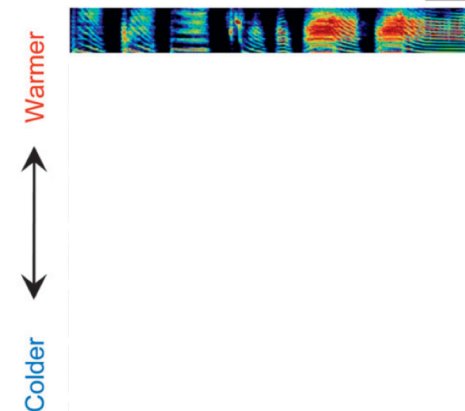
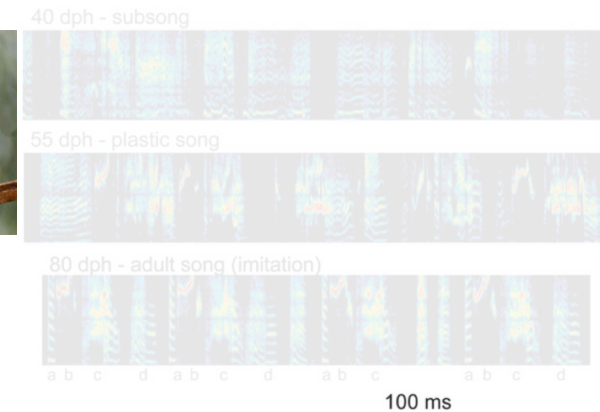
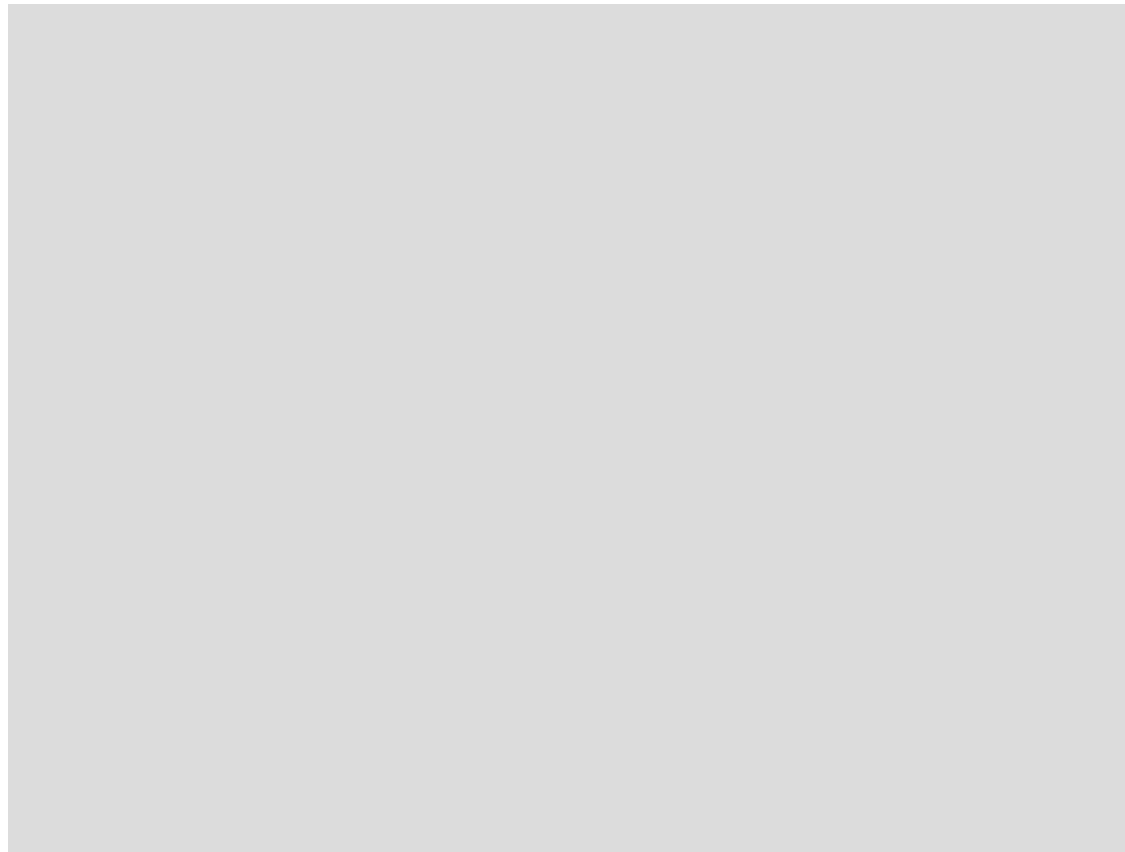
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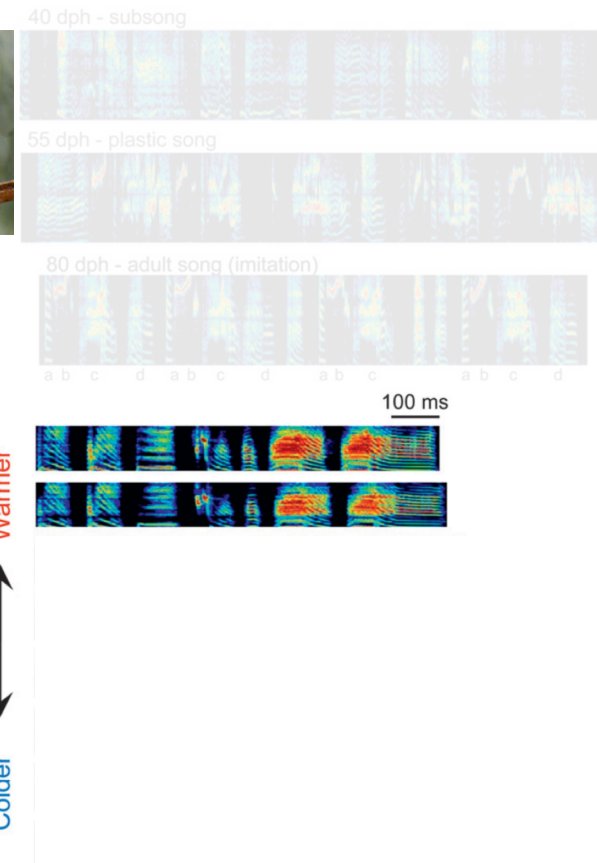
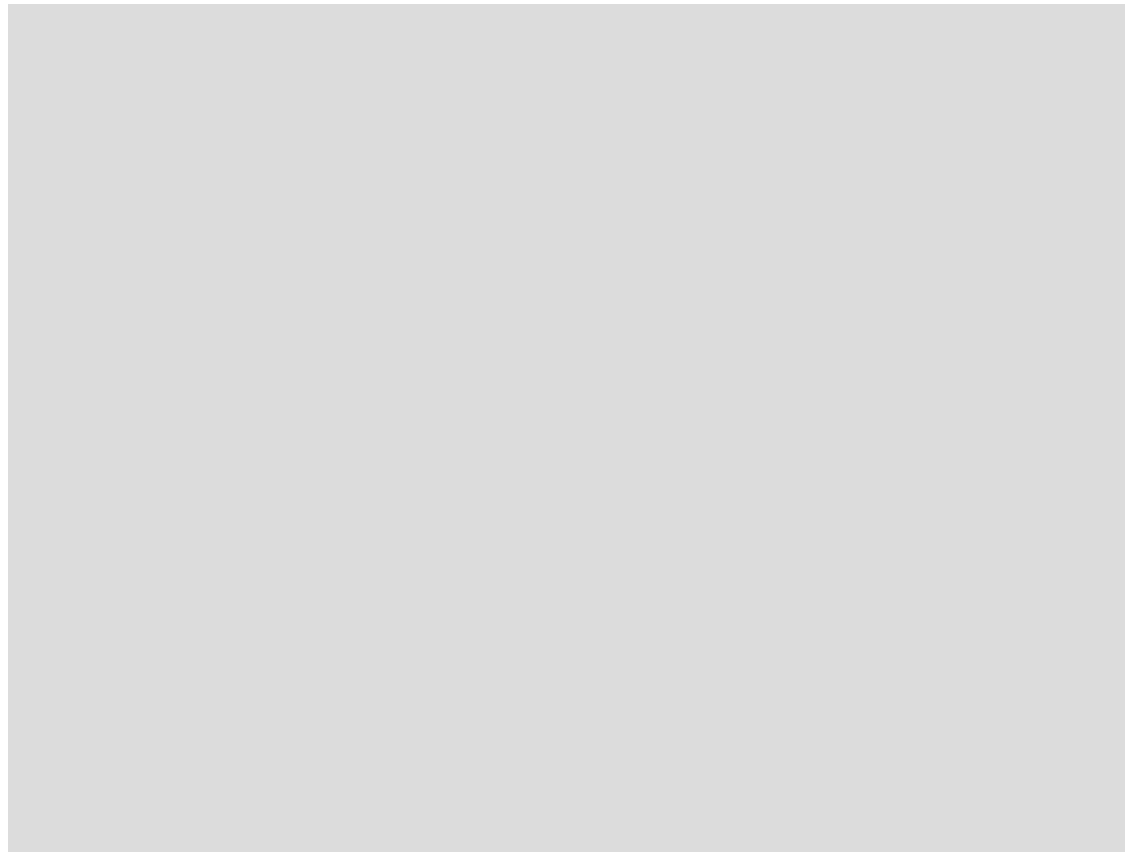
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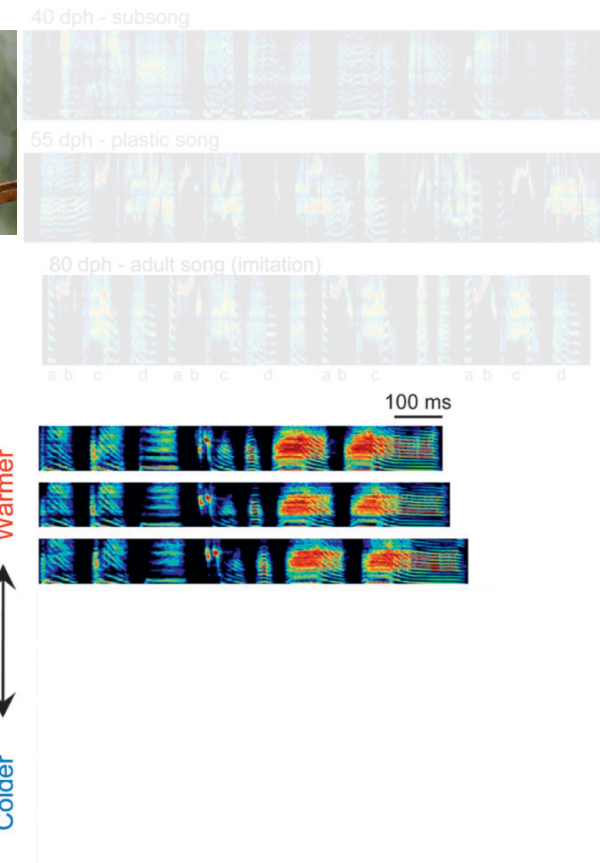
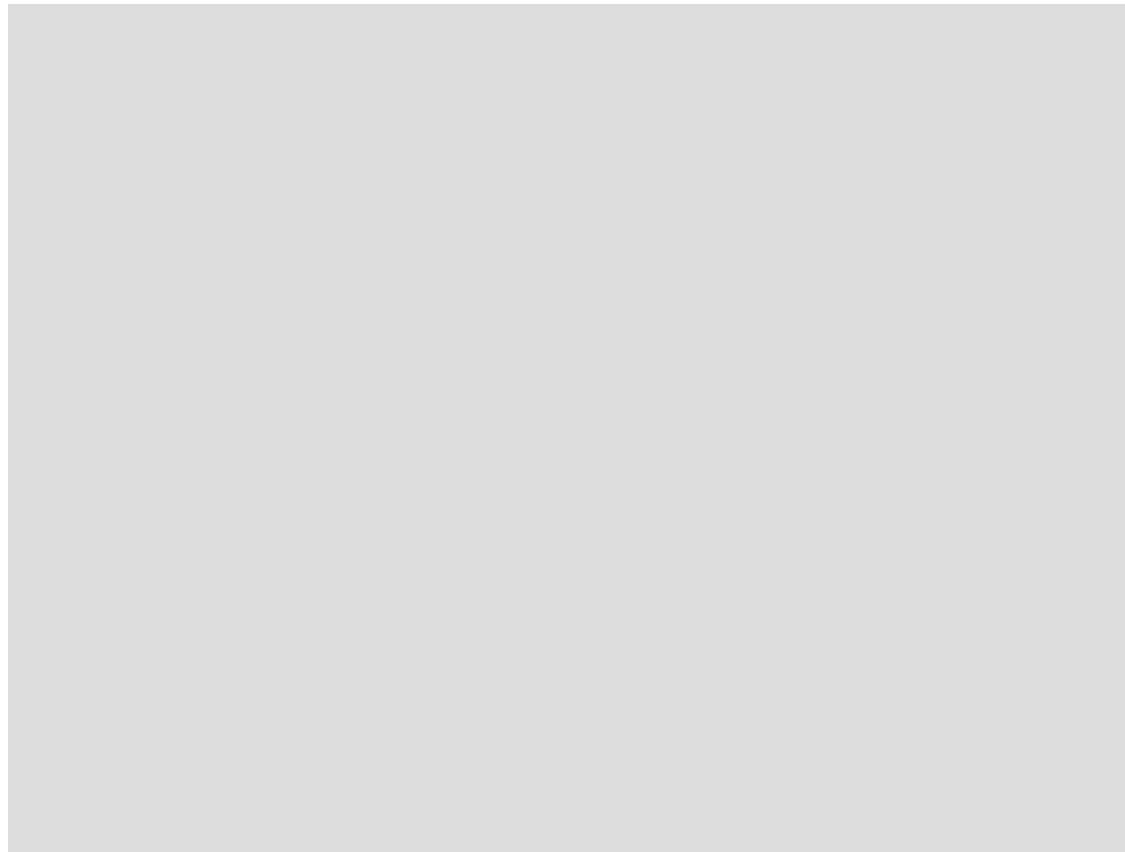
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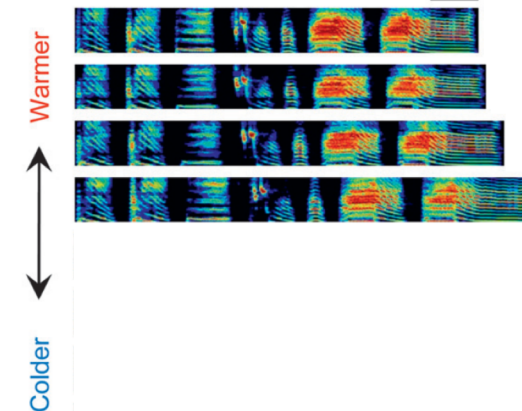
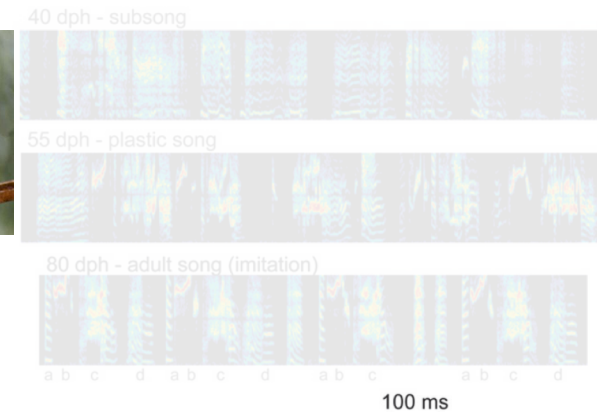
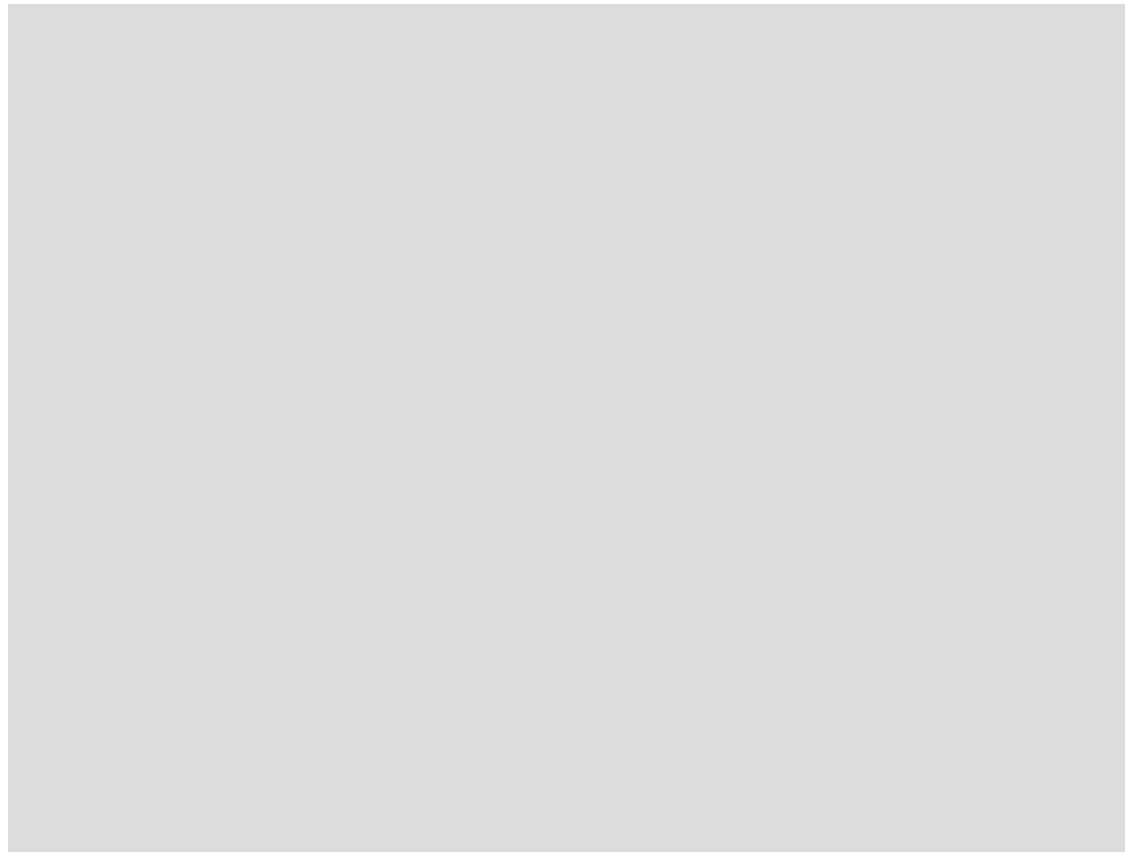
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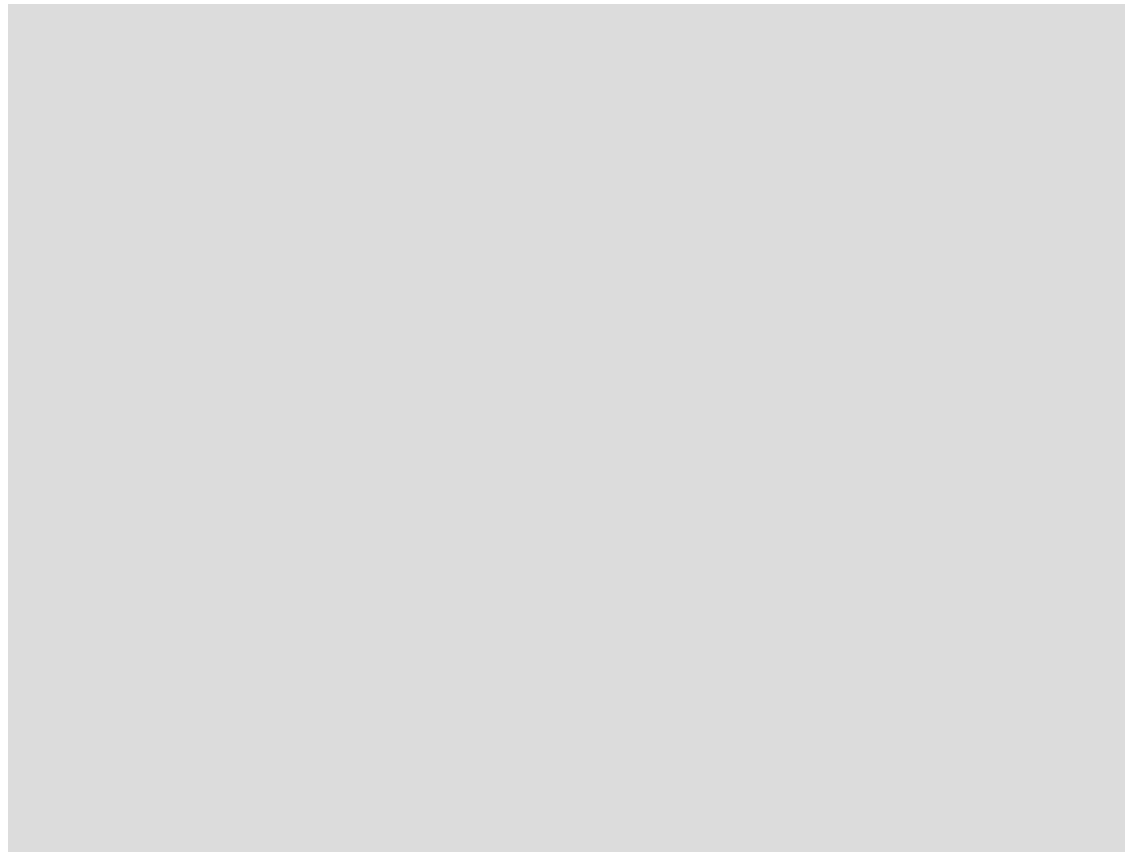
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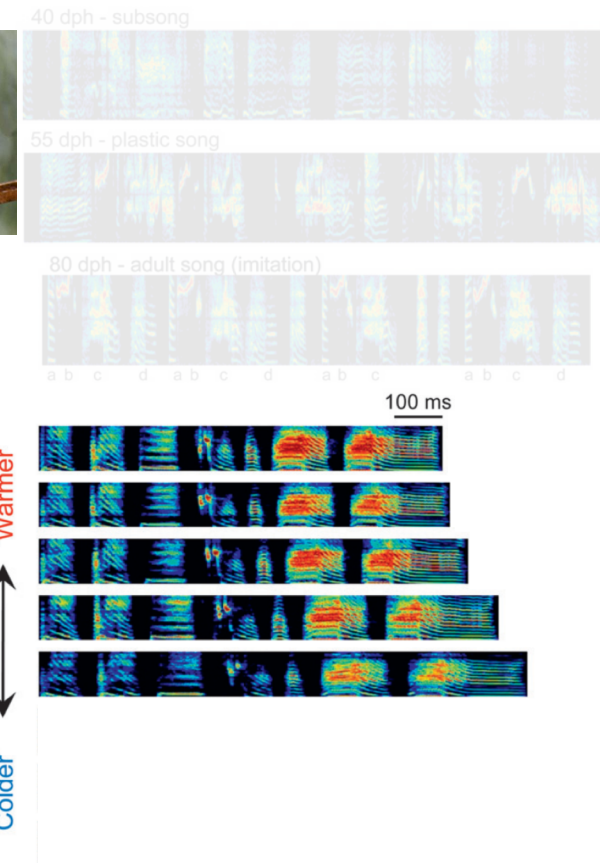


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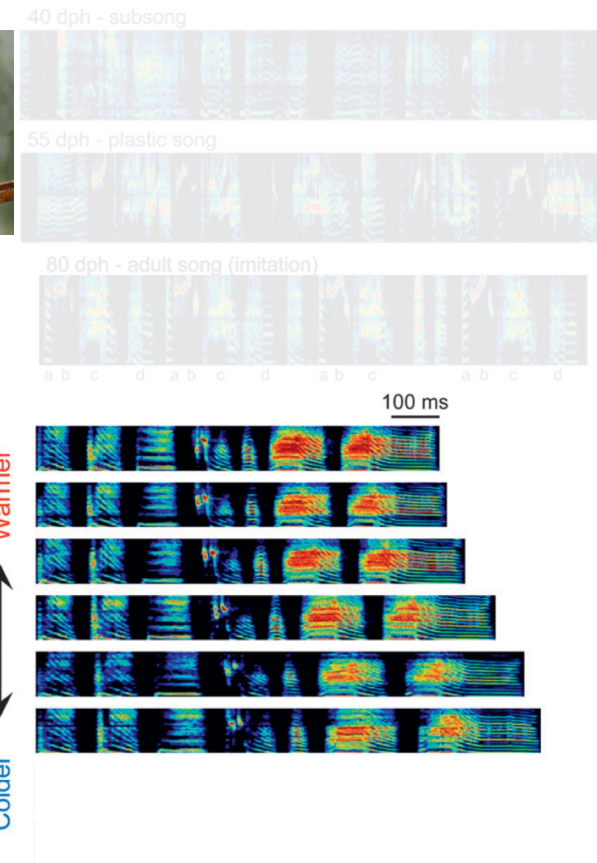
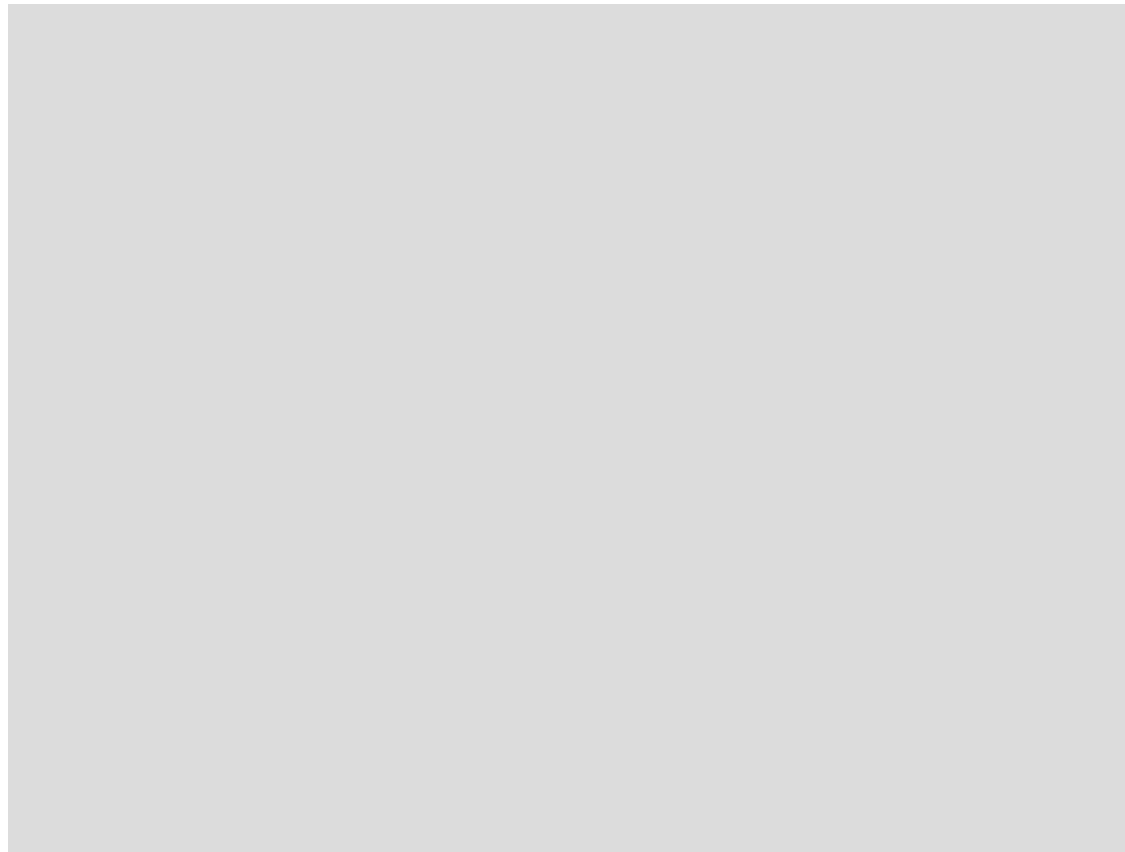
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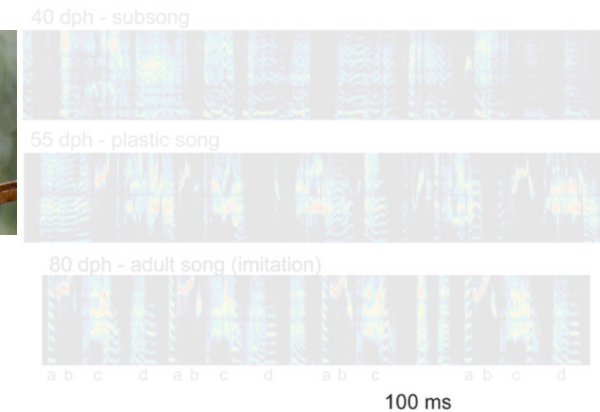
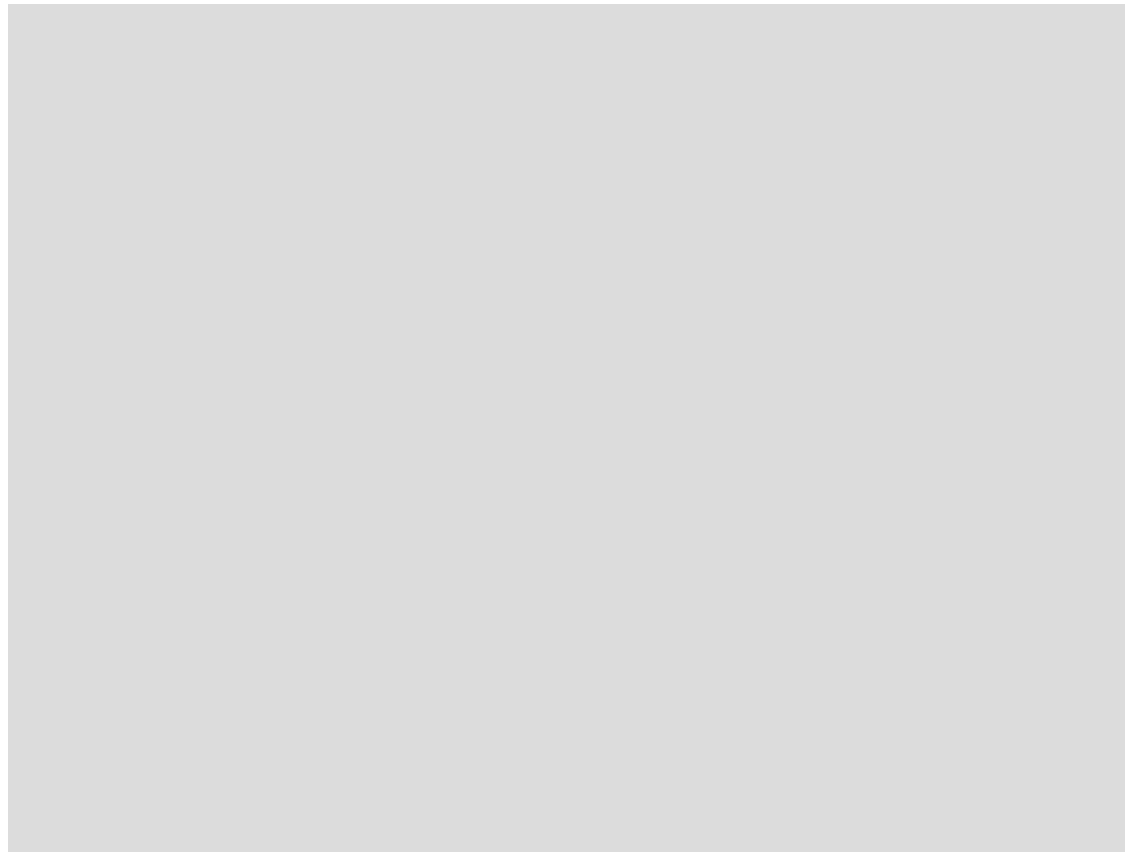
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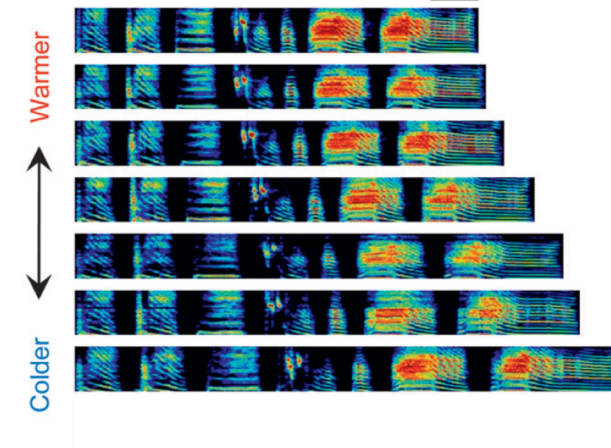
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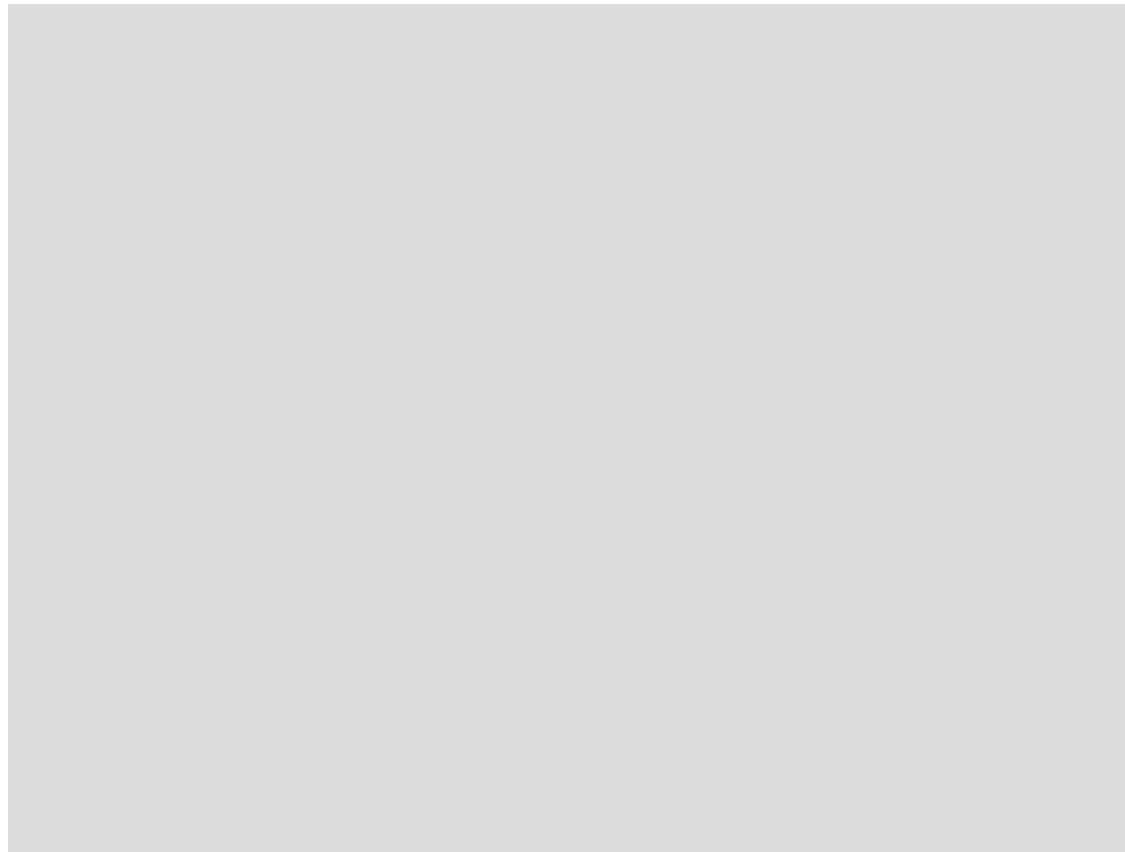
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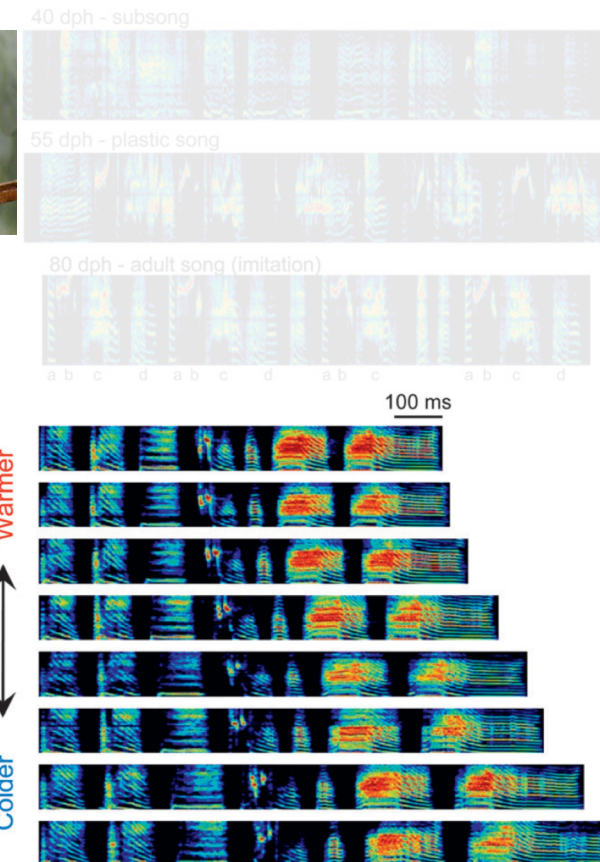
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# Computers are engineered, neural systems are trained

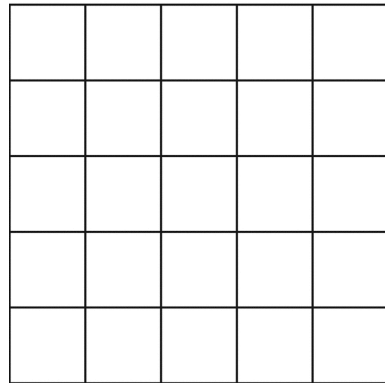
ial

solely by observing examples?

# Computers are engineered, neural systems are trained

- Computers: binary memory

ial

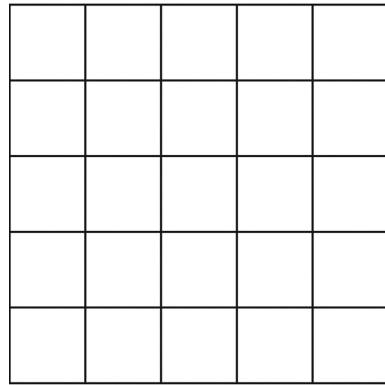


solely by observing examples?

# Computers are engineered, neural systems are trained

- Computers: binary memory
- Task:  $7 + 17$

al



solely by observing examples?

# Computers are engineered, neural systems are trained

- Computers: binary memory
- Task:  $7 + 17$ 
  - Encode: decimal  $\Rightarrow$  binary

al

7	→	0	0	1	1	1
17	→	1	0	0	0	1

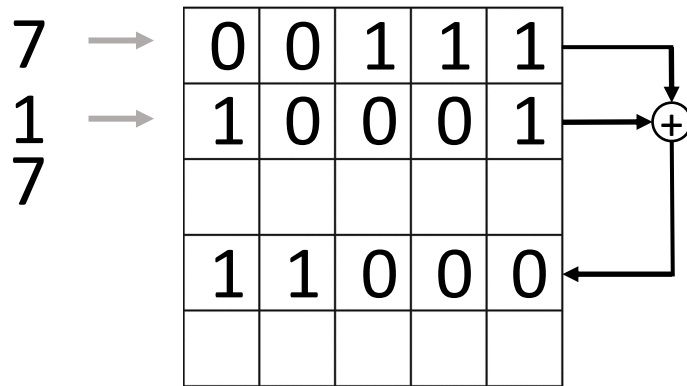
solely by observing examples?



# Computers are engineered, neural systems are trained

- Computers: binary memory
- Task:  $7 + 17$ 
  - Encode: decimal  $\Rightarrow$  binary
  - Modify: operations in binary

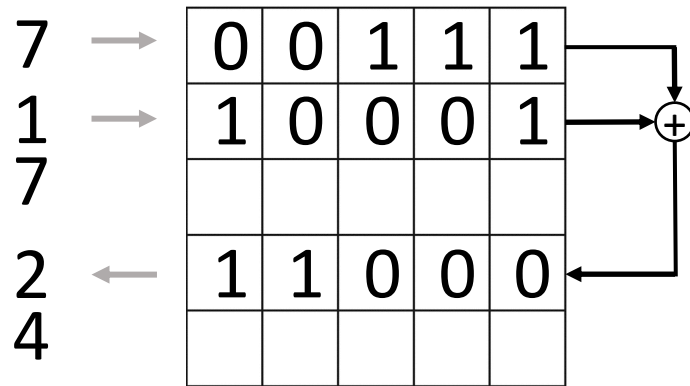
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# Computers are engineered, neural systems are trained

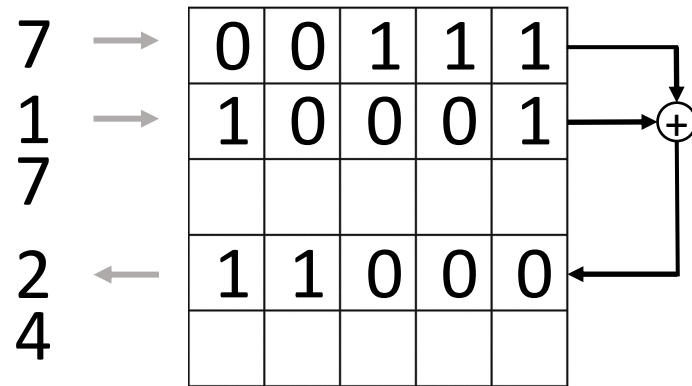
- Computers: binary memory
- Task:  $7 + 17$ 
  - Encode: decimal  $\Rightarrow$  binary
  - Modify: operations in binary
  - Decode: binary  $\Rightarrow$  decimal



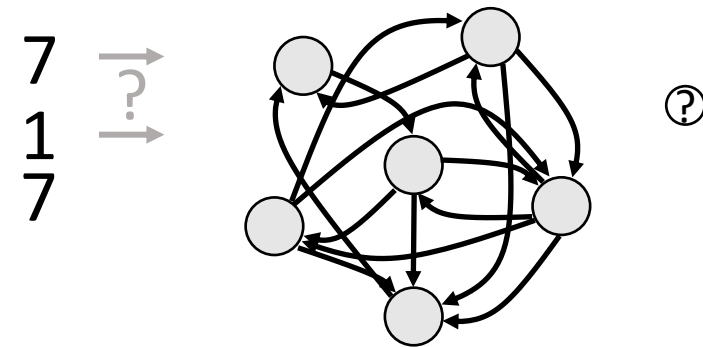
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# Computers are engineered, neural systems are trained

- Computers: binary memory
- Task:  $7 + 17$ 
  - Encode: decimal  $\Rightarrow$  binary
  - Modify: operations in binary
  - Decode: binary  $\Rightarrow$  decimal



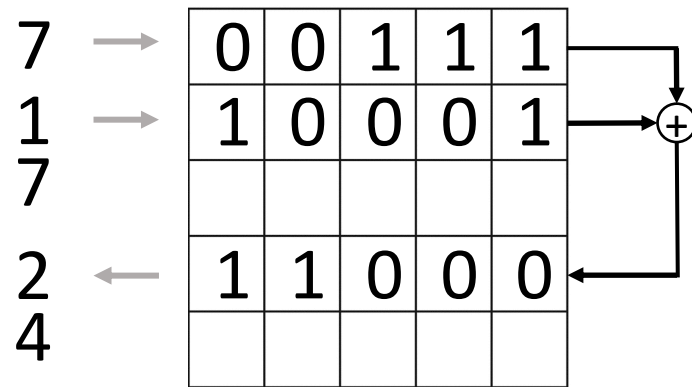
- Brain: neural memory
- Task:  $7 + 17$ 
  - Encode: ?
  - Modify: ?
  - Decode: ?



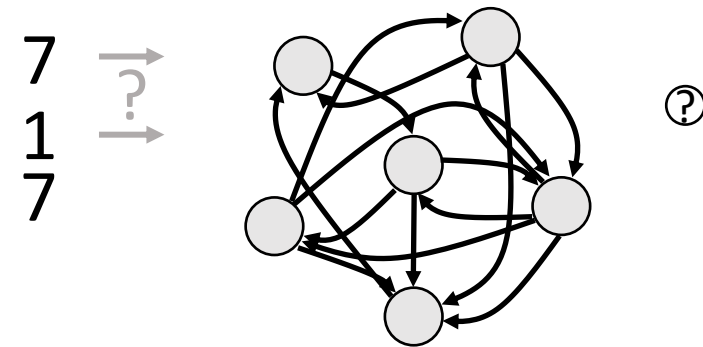
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# Computers are engineered, neural systems are trained

- Computers: binary memory
- Task:  $7 + 17$ 
  - Encode: decimal  $\Rightarrow$  binary
  - Modify: operations in binary
  - Decode: binary  $\Rightarrow$  decimal

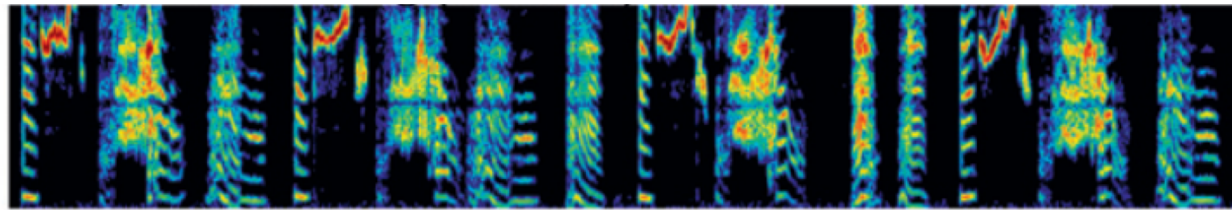


- Brain: neural memory
- Task:  $7 + 17$ 
  - Encode: ?
  - Modify: ?
  - Decode: ?



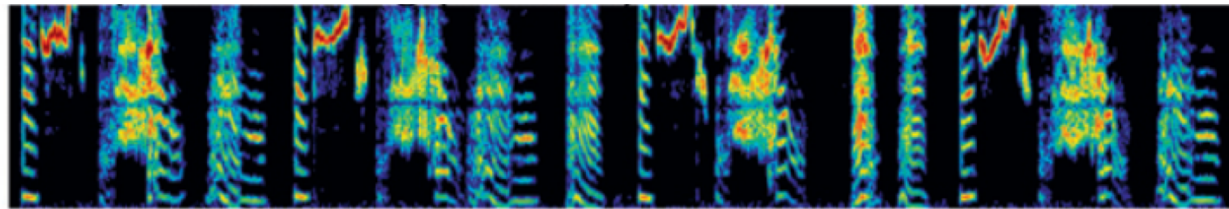
**How can neural systems learn to manipulate information solely by observing examples?**

# Model of memory



# Unpredictable yet structured data

- Temporal (changes with time)
- Structured (not completely random)
- Complex (unpredictable)



# Unpredictable yet structured data

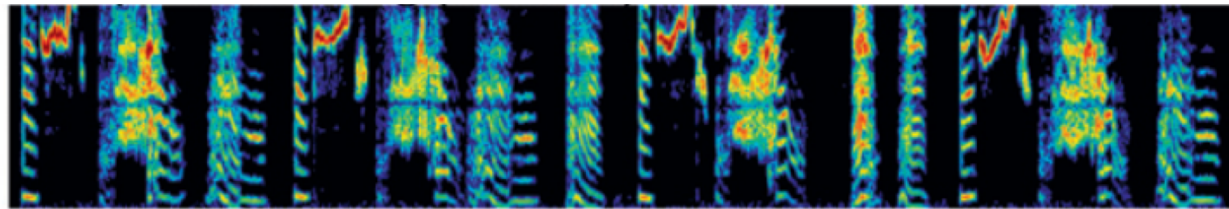
- Temporal (changes with time)
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Lorenz  
Attractor

$$\dot{x}_1 = \rho(x_2 - x_1)$$

$$\dot{x}_2 = x_1(\rho - x_3) - x_2$$

$$\dot{x}_3 = x_1x_2 - \beta x_3,$$



# Unpredictable yet structured data

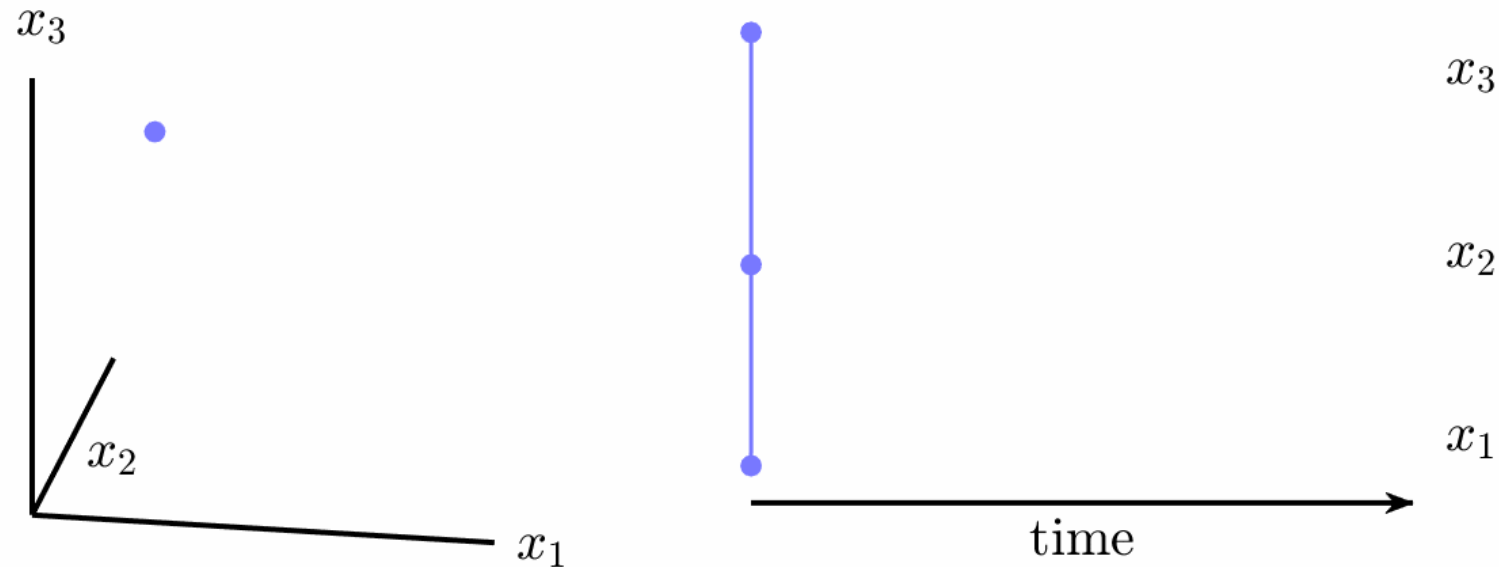
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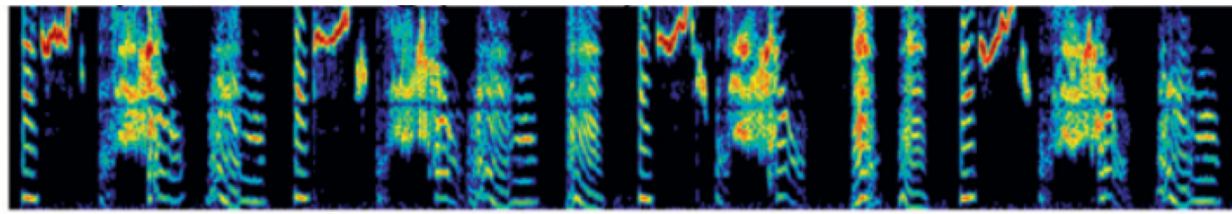
$$\dot{x}_2 = x_1(\rho - x_3) - x_2$$

$$\dot{x}_3 = x_1x_2 - \beta x_3,$$

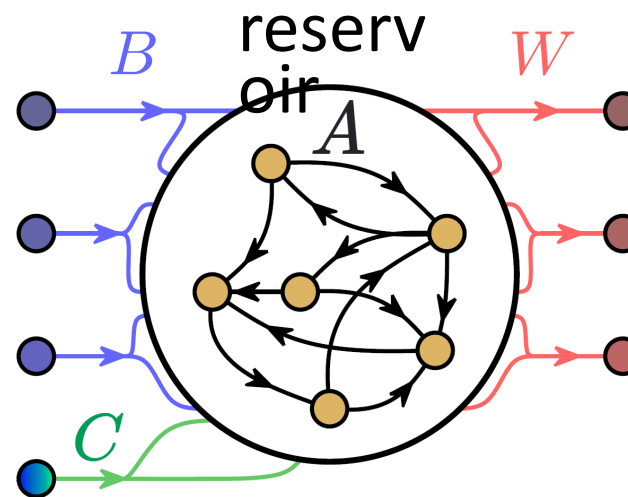




# Model of neural network

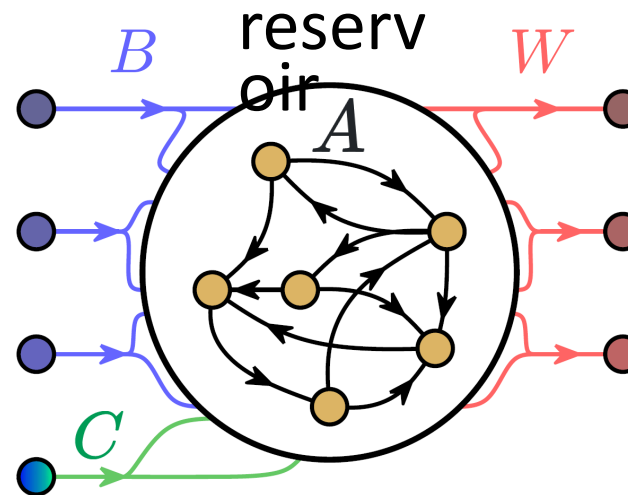


# Recurrent neural network (reservoir computer)



# Recurrent neural network (reservoir computer)

$$\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{C}\mathbf{c} + \mathbf{d})$$

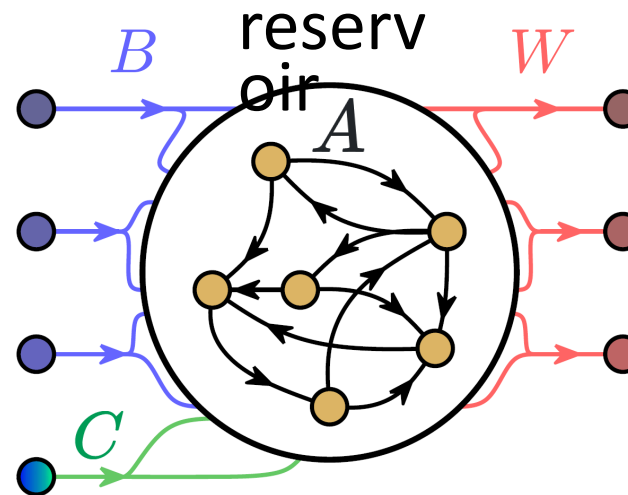


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$$\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{C}\mathbf{c} + \mathbf{d})$$

Diagram illustrating the equation above, with arrows pointing from the terms to their corresponding components in the diagram below:

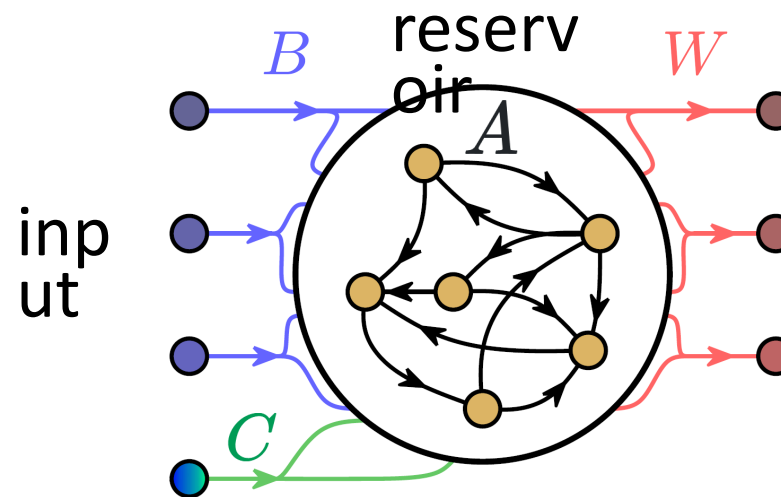
- $\mathbf{r}$  (reservoir state) points to the central reservoir node.
- $\mathbf{A}$  (reservoir matrix) points to the internal connections within the reservoir.
- $\mathbf{B}$  (input matrix) points to the input connections from the left.
- $\mathbf{C}$  (control matrix) points to the control input connection from the bottom.
- $\mathbf{d}$  (bias) points to the bias input connection from the bottom.



# Recurrent neural network (reservoir computer)

$$\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{C}\mathbf{c} + \mathbf{d})$$

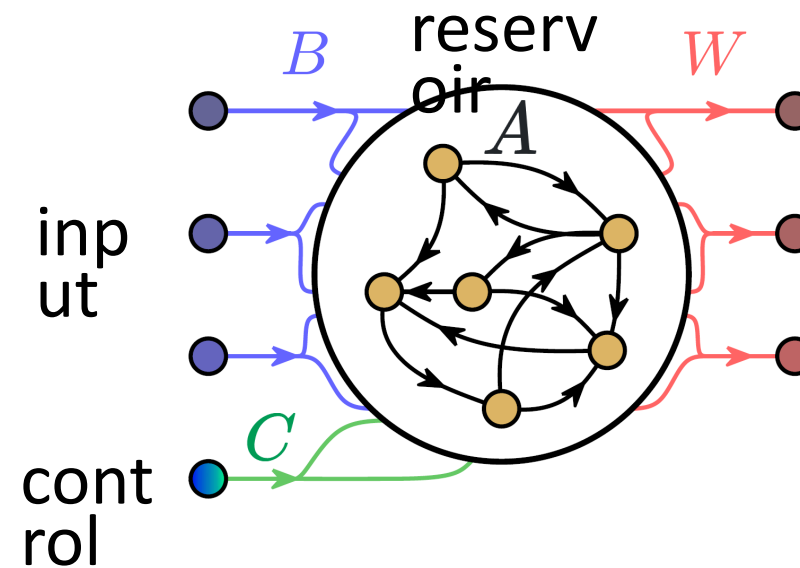
Diagram illustrating the equation above, with arrows pointing to the terms:  $\mathbf{r}$  (reservoir state),  $\mathbf{A}$  (reservoir matrix),  $\mathbf{B}$  (input matrix),  $\mathbf{x}$  (input vector),  $\mathbf{C}$  (bias matrix), and  $\mathbf{d}$  (bias vector).



# Recurrent neural network (reservoir computer)

$$\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{C}\mathbf{c} + \mathbf{d})$$

Diagram illustrating the equation above, with arrows pointing to the terms:  $\mathbf{r}$  (reserv),  $\mathbf{A}\mathbf{r}$  (oir),  $\mathbf{B}\mathbf{x}$  (inp ut), and  $\mathbf{C}\mathbf{c}$  (cont rol).

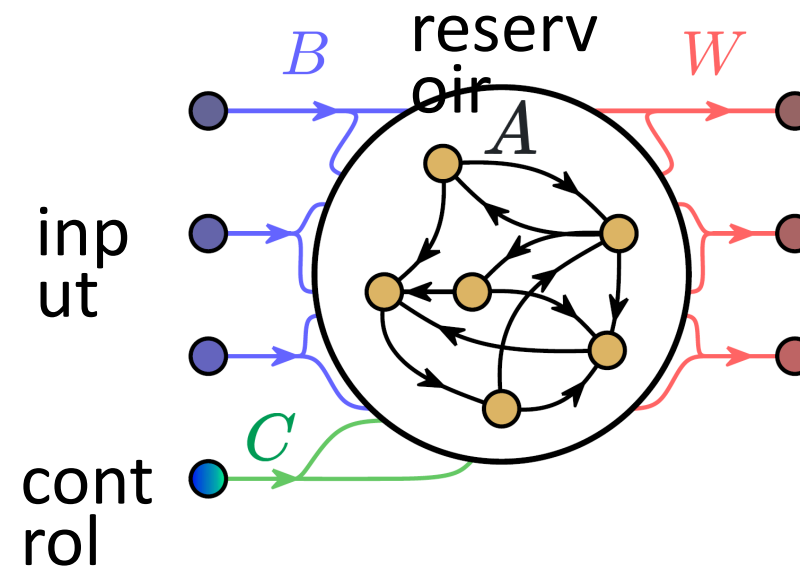


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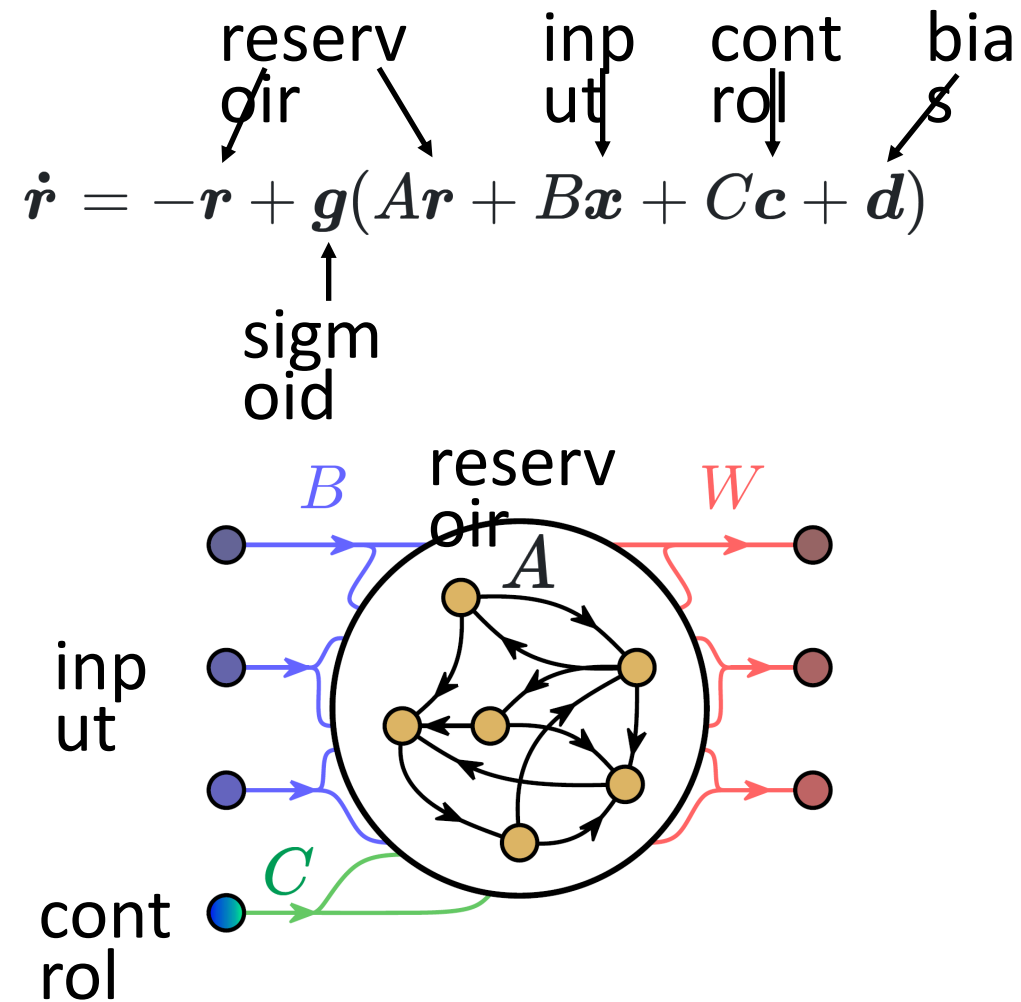
$$\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{C}\mathbf{c} + \mathbf{d})$$

Diagram illustrating the equation components with arrows pointing to the terms in the equation:

- $\mathbf{r}$  (reservoir state) is labeled "reserv" and "oir".
- $\mathbf{A}$  (reservoir matrix) is labeled "oir".
- $\mathbf{x}$  (input vector) is labeled "inp" and "ut".
- $\mathbf{c}$  (control vector) is labeled "cont" and "rol".
- $\mathbf{d}$  (bias vector) is labeled "bia".

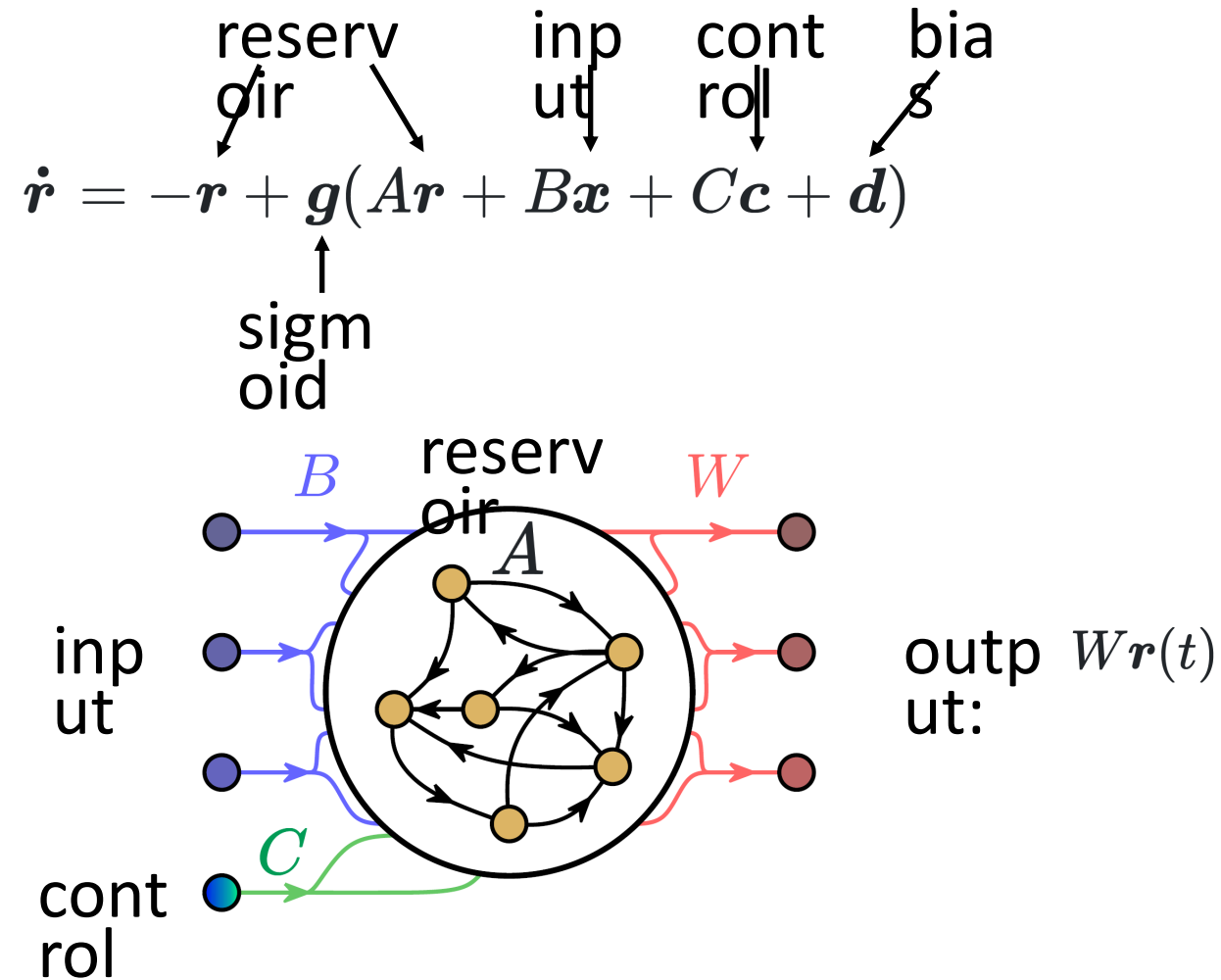


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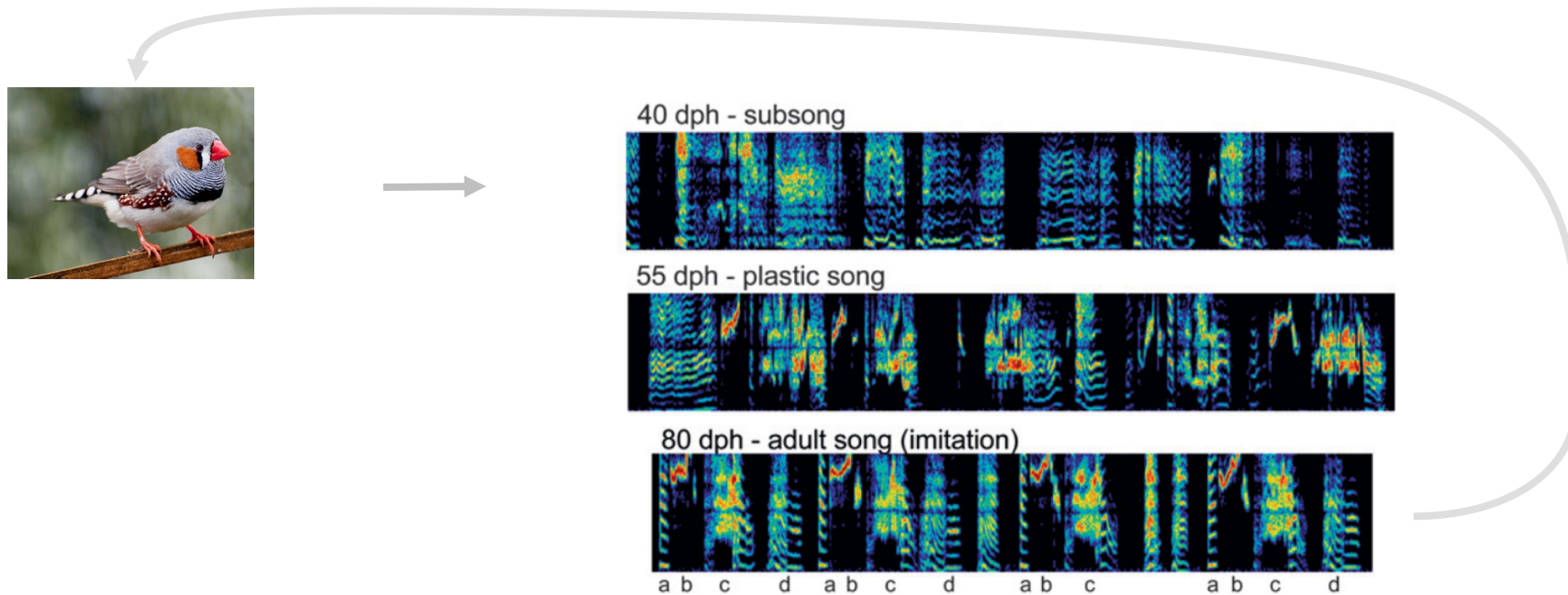




# Recurrent neural network (reservoir computer)



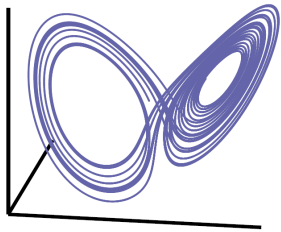
# Learning memories by imitating examples



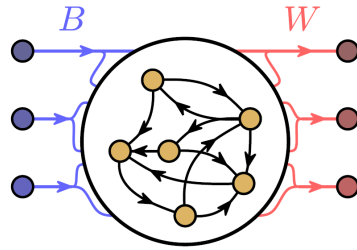
# RNNs sustain chaotic memories by imitating examples

$$\text{Driving } \dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{d})$$

Lorenz attractor



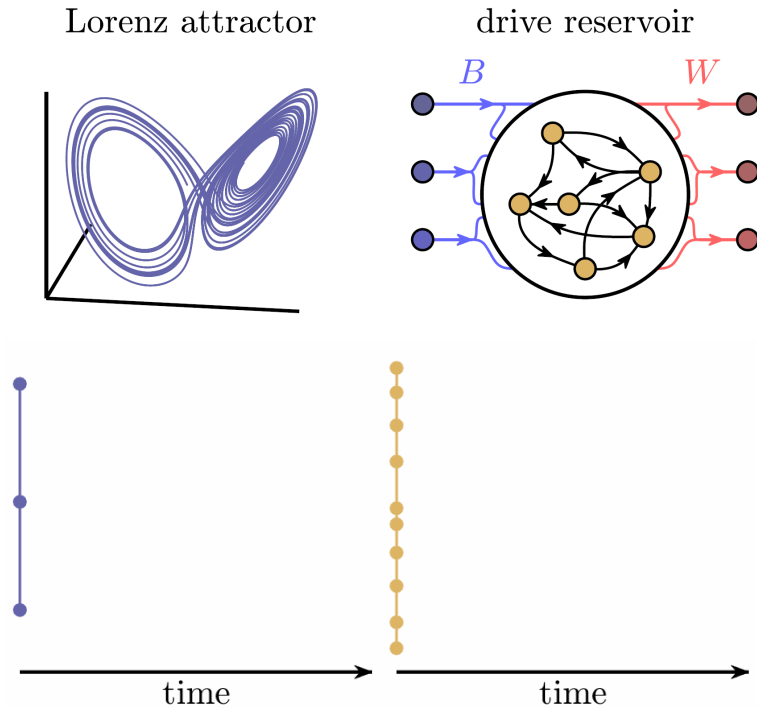
drive reservoir



Sussillo, D., & Abbott, L. F. (2009). Generating Coherent Patterns of Activity from Chaotic Neural Networks. *Neuron*, 63(4), 544–557.  
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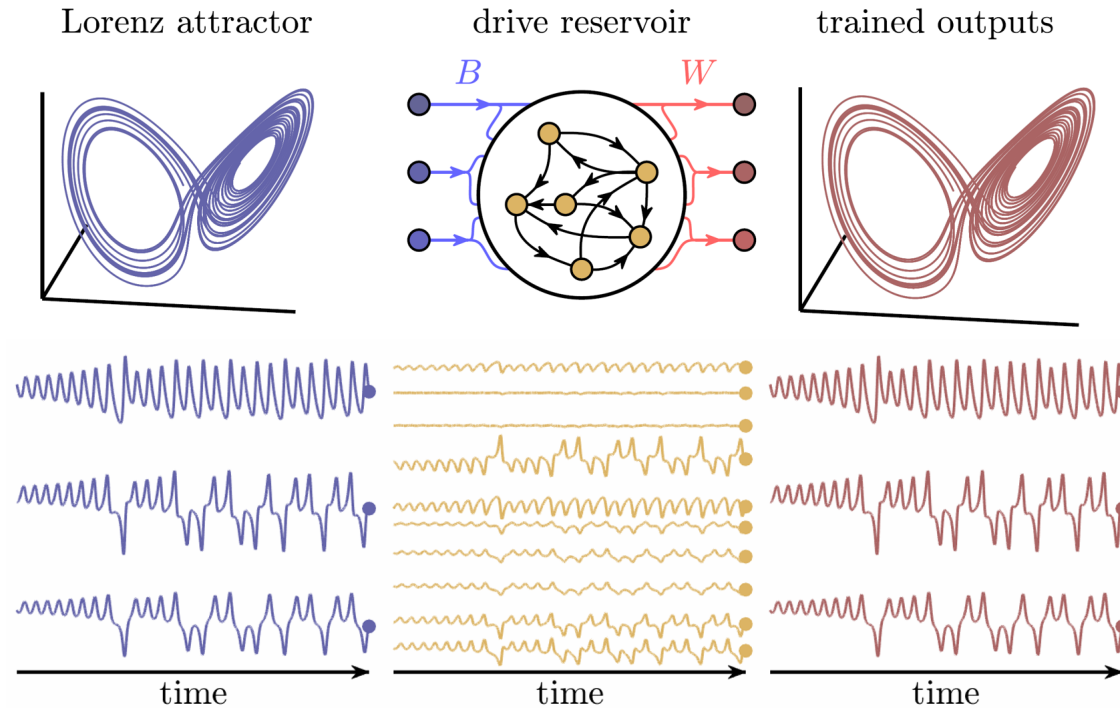


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Jaeger, H. (2010). The “echo state” approach to analysing and training recurrent neural networks – with an Erratum note. *GMD Report*, 1(148), 1–47.

# RNNs sustain chaotic memories by imitating examples

Driving  $\dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{d}) \rightarrow$  Trainin

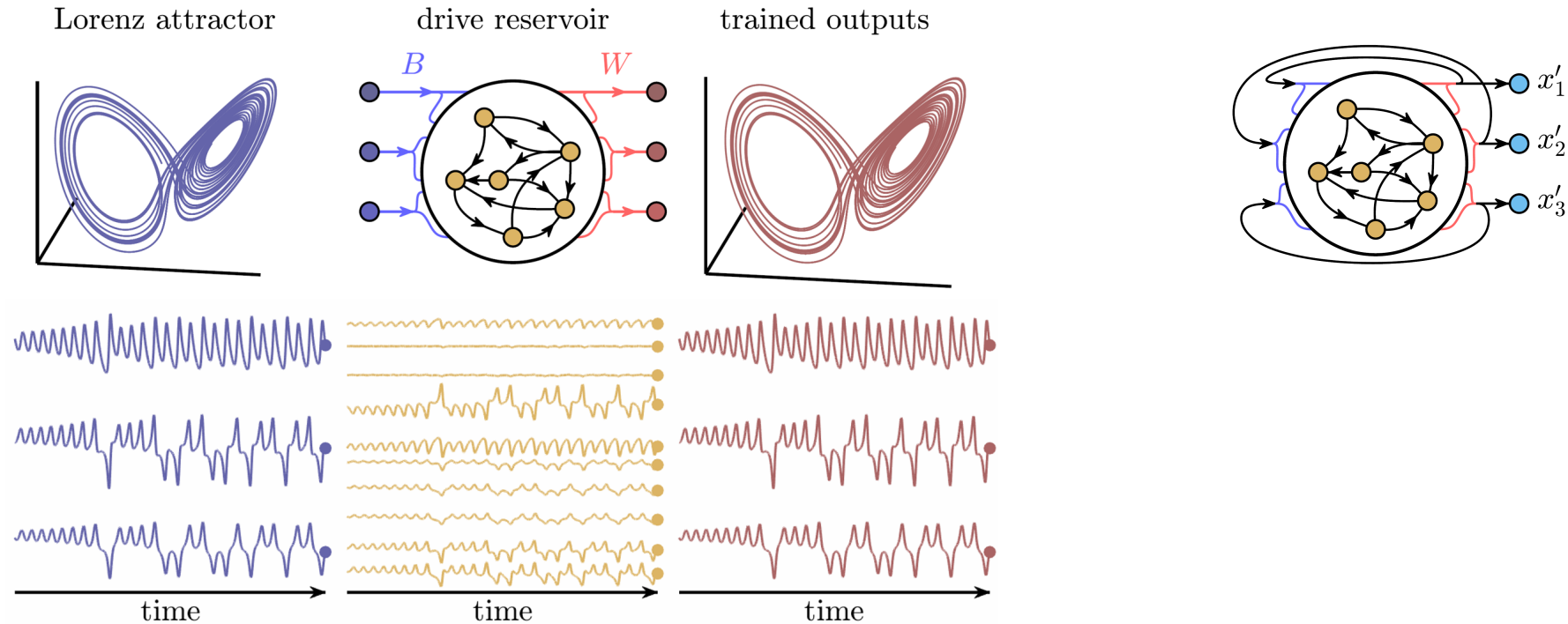


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$$\text{Driving } \dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}(\mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{x} + \mathbf{d}) \rightarrow \text{Training } \rightarrow \mathbf{W} \quad \text{Pr } \dot{\mathbf{r}} = -\mathbf{r} + \mathbf{g}([\mathbf{A} + \mathbf{B}\mathbf{W}]\mathbf{r} + \mathbf{d})$$

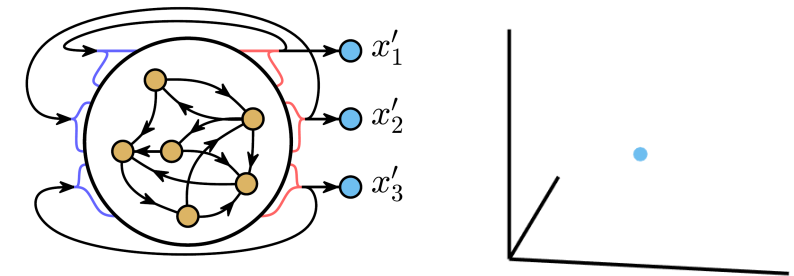
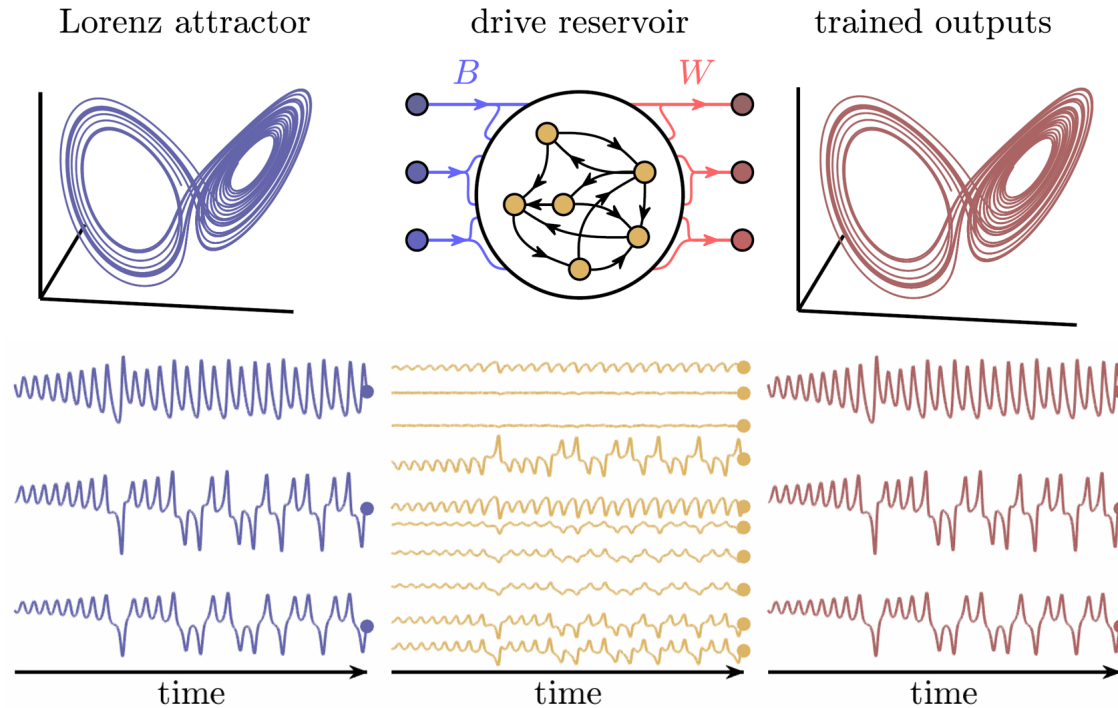


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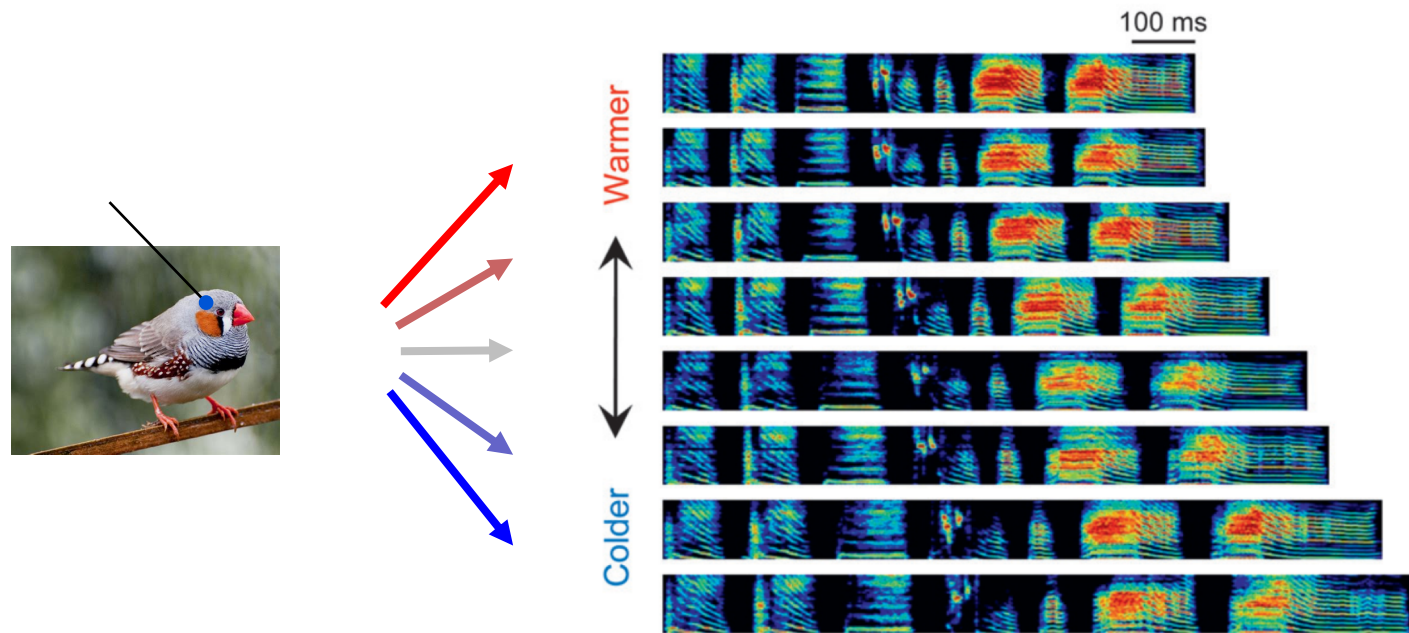


No engineered encoding or decoding!

Sussillo, D., & Abbott, L. F. (2009). Generating Coherent Patterns of Activity from Chaotic Neural Networks. *Neuron*, 63(4), 544–557.  
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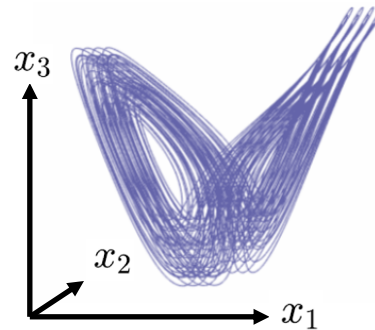
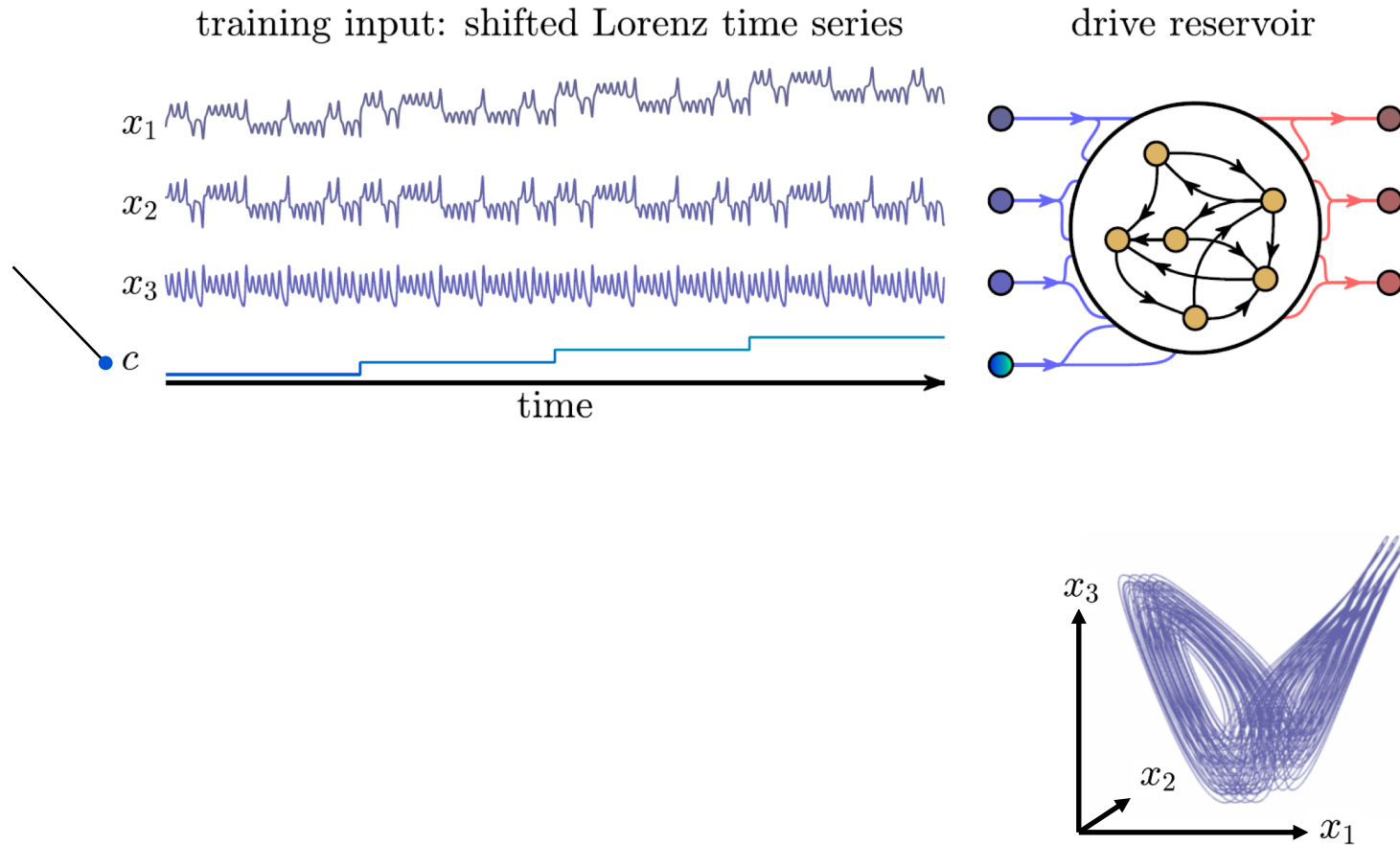


# Learning computations by imitating examples



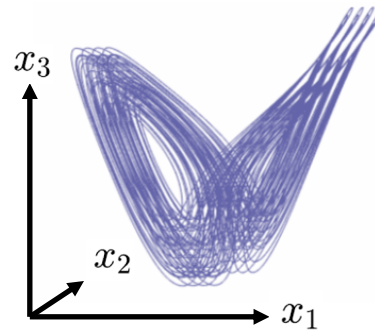
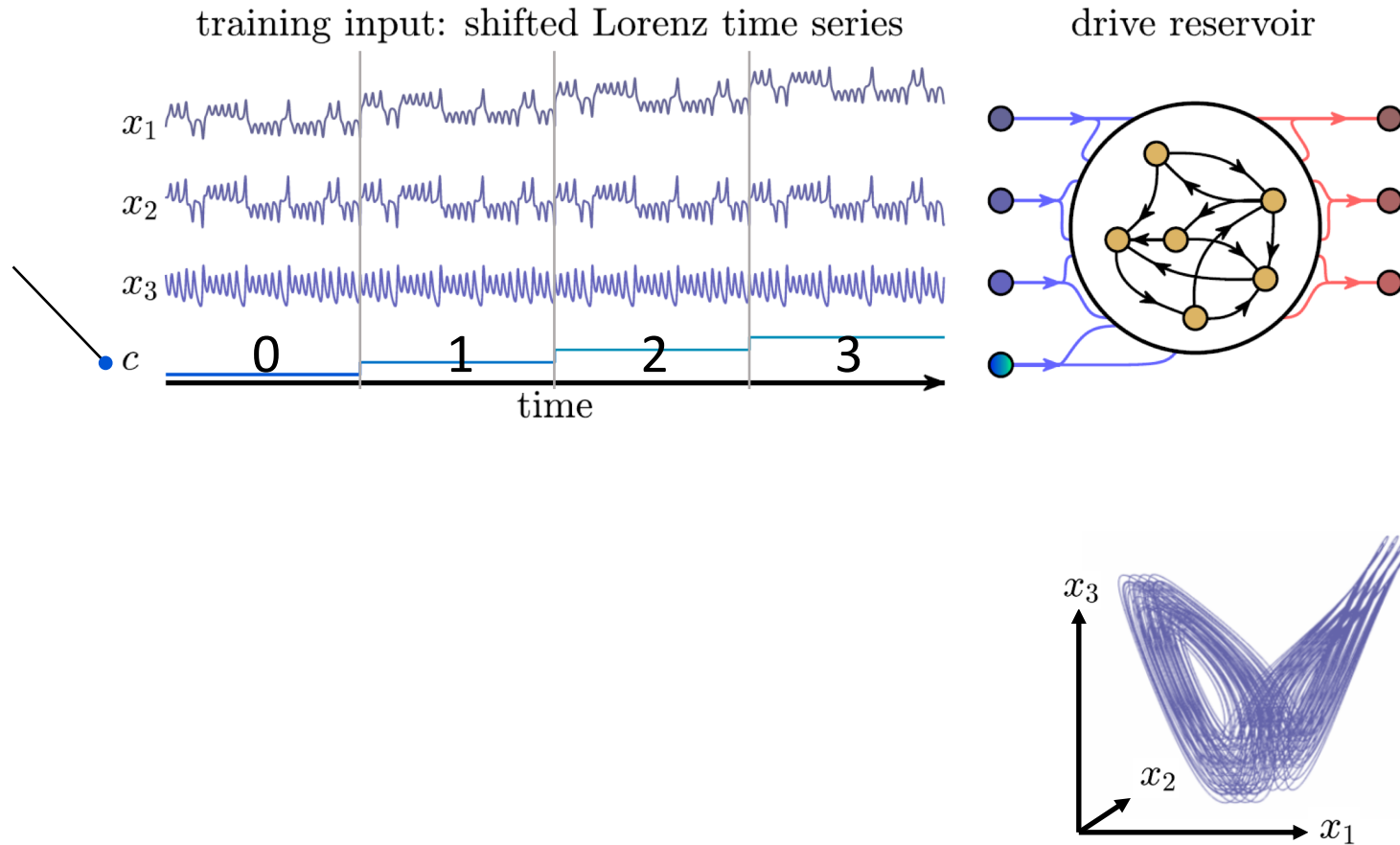


# RNNs translate chaotic memories by imitating examples



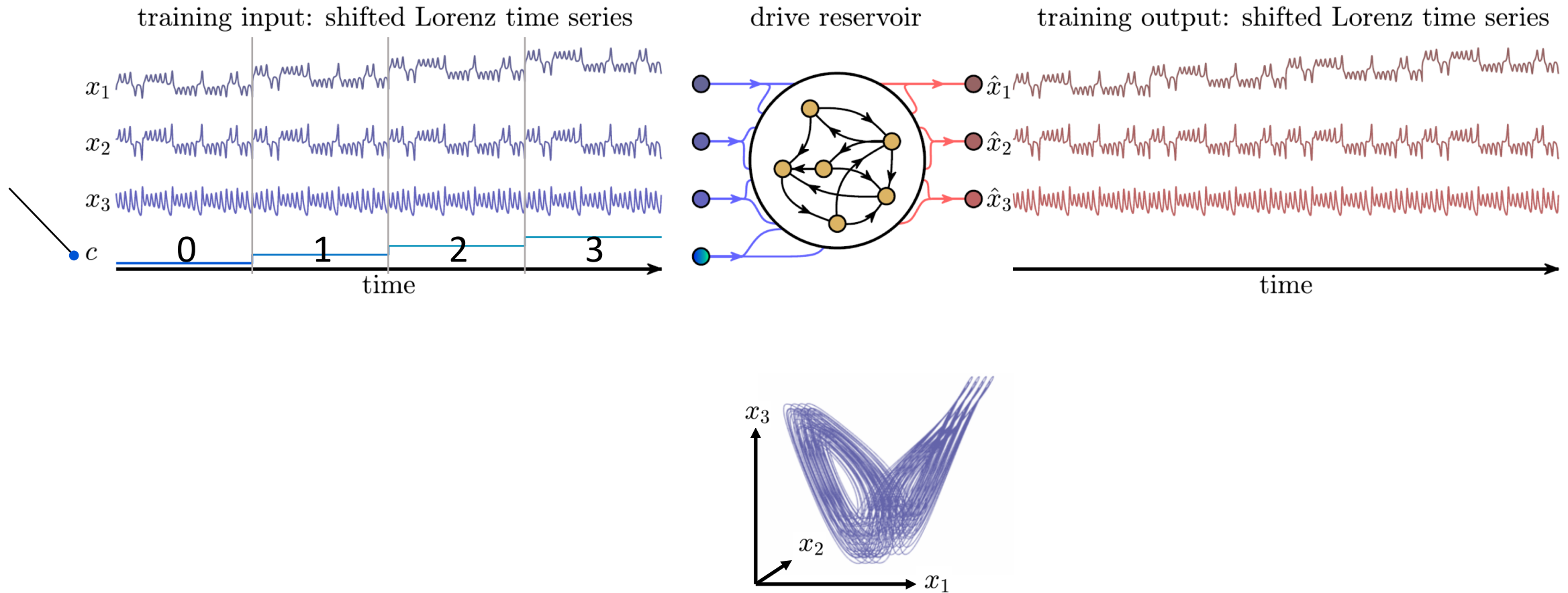
Kim, J. Z., Lu, Z., Nozari, E., Pappas, G. J., & Bassett, D. S. (2020). Teaching Recurrent Neural Networks to Model Chaotic Memories by Example. (Accepted, *Nat. Mach. Intell.*).

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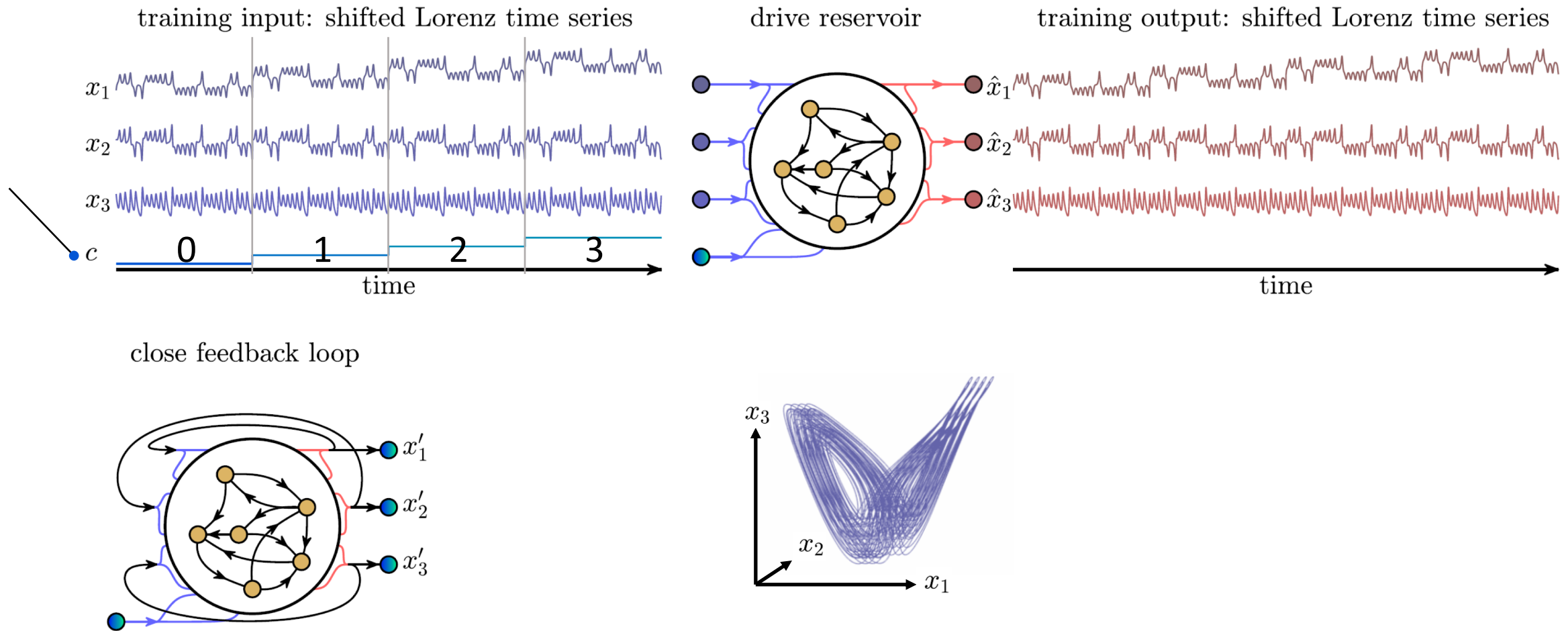
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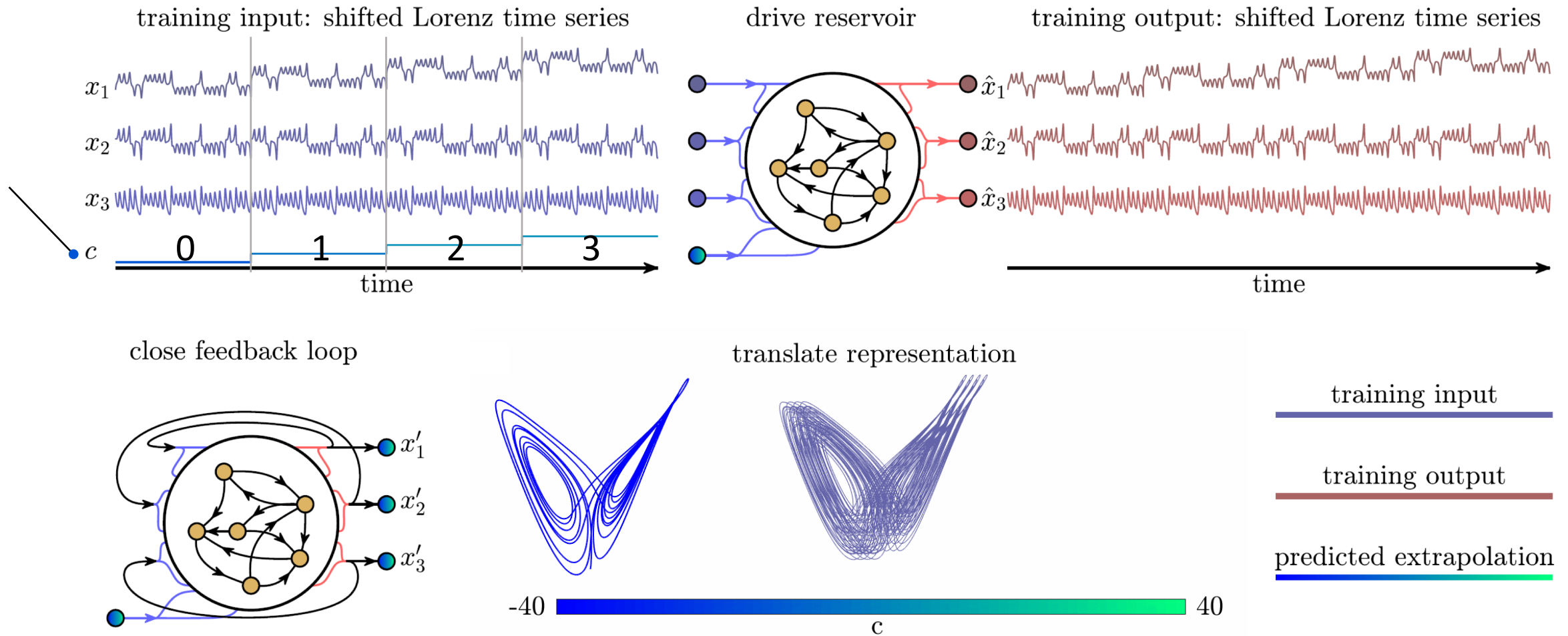
Kim, J. Z., Lu, Z., Nozari, E., Pappas, G. J., & Bassett, D. S. (2020). Teaching Recurrent Neural Networks to Modify Chaotic Memories by Example. (Accepted, *Nat. Mach. Intell.*).

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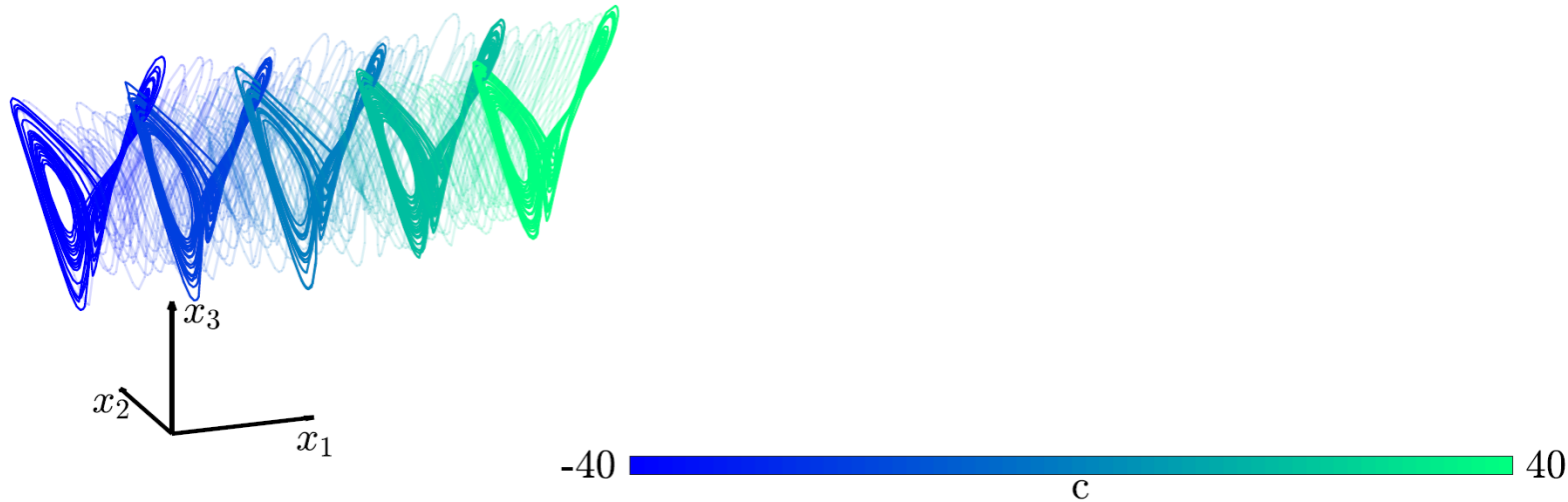
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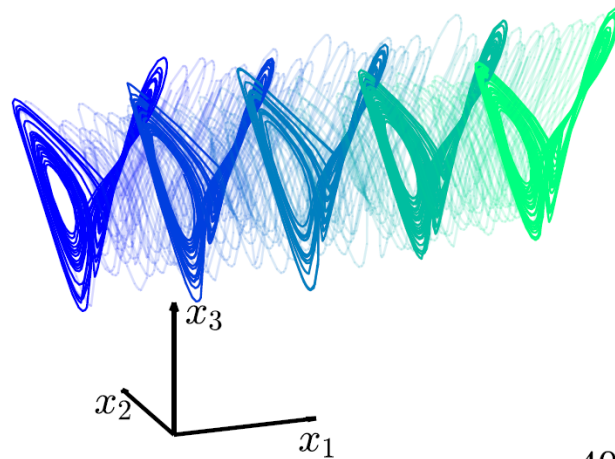
# RNNs translate chaotic memories by imitating examples

translate  $x_1$

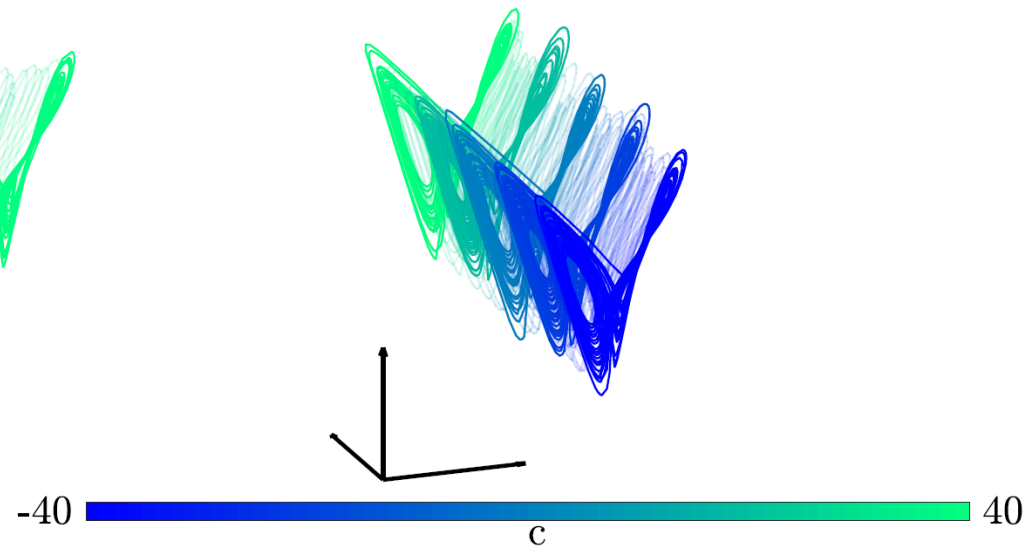


# RNNs translate chaotic memories by imitating examples

translate  $x_1$



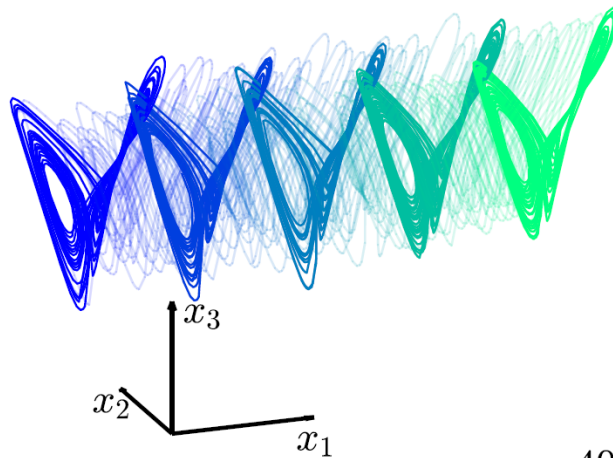
translate  $x_2$



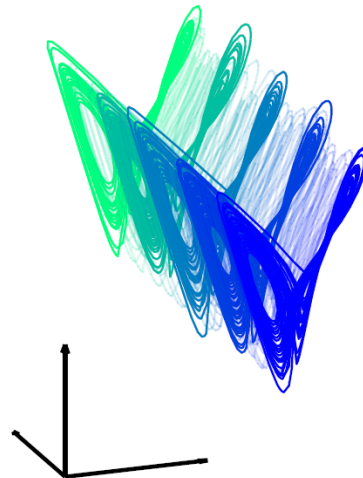


# RNNs translate chaotic memories by imitating examples

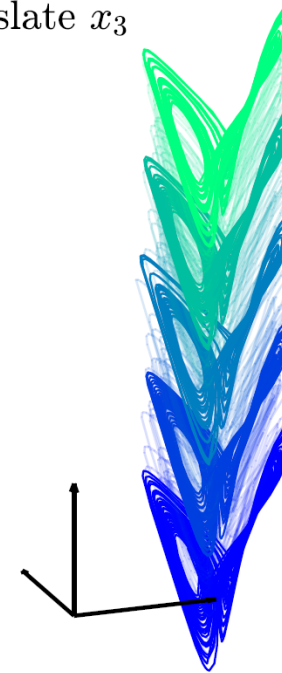
translate  $x_1$



translate  $x_2$



translate  $x_3$





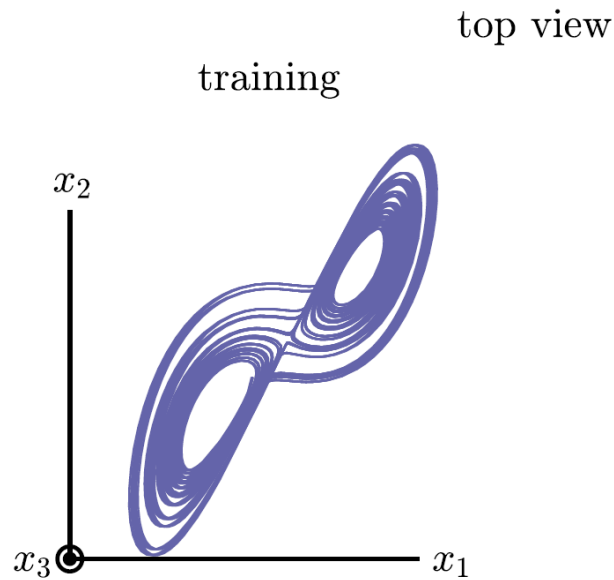
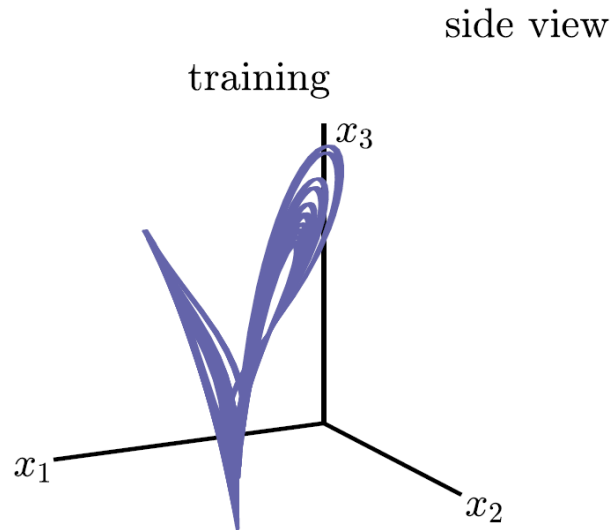
# RNNs transform chaotic memories by imitating examples

# RNNs transform chaotic memories by imitating examples

- Can we change the actual geometry of the attractor manifold?

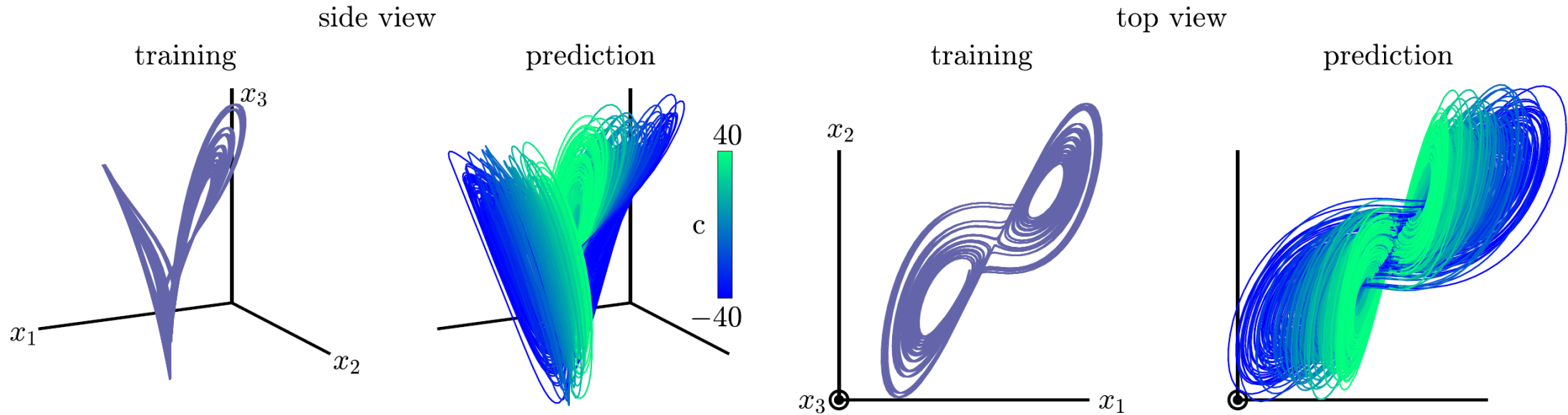
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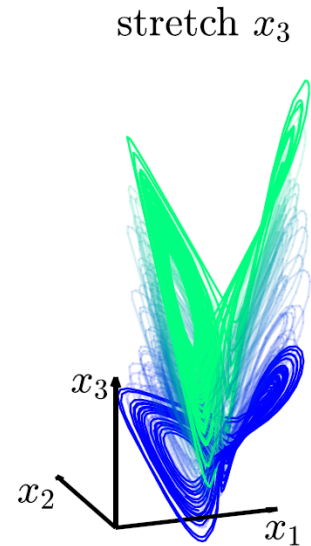


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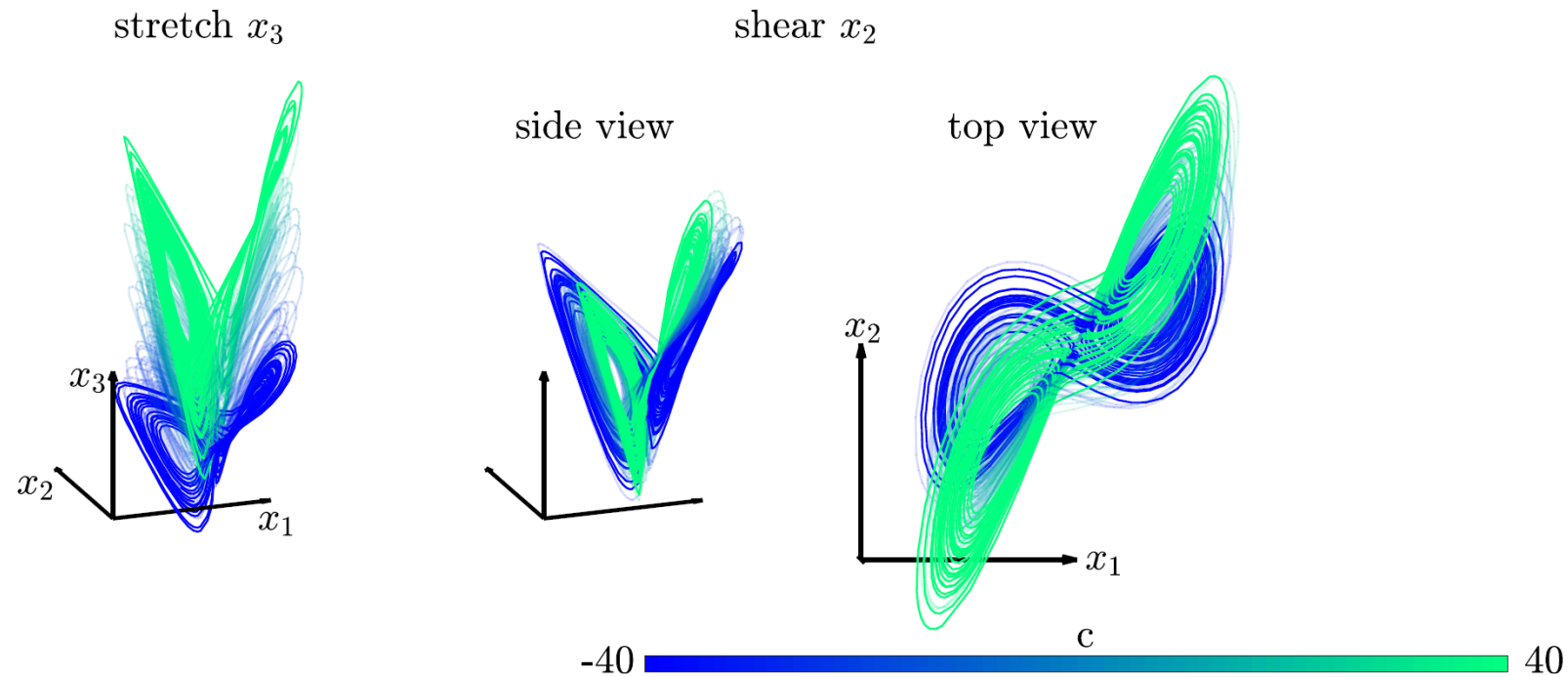
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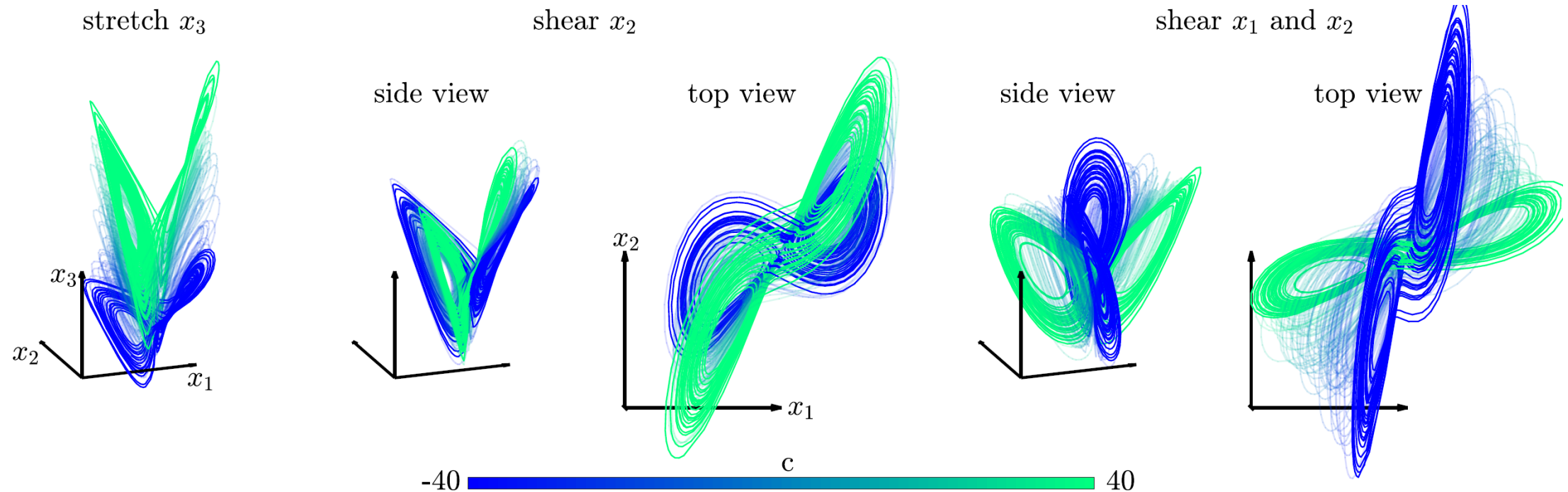
# RNNs transform chaotic memories by imitating examples



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# RNNs predict highly nonlinear events by imitating examples



# RNNs predict highly nonlinear events by imitating examples

- The Lorenz attractor undergoes a subcritical Hopf bifurcation
  - Fixed points at the wings lose stability

Lorenz

system:

$$\dot{x}_1 = \rho(x_2 - x_1)$$

$$\dot{x}_2 = x_1(\rho - x_3) - x_2$$

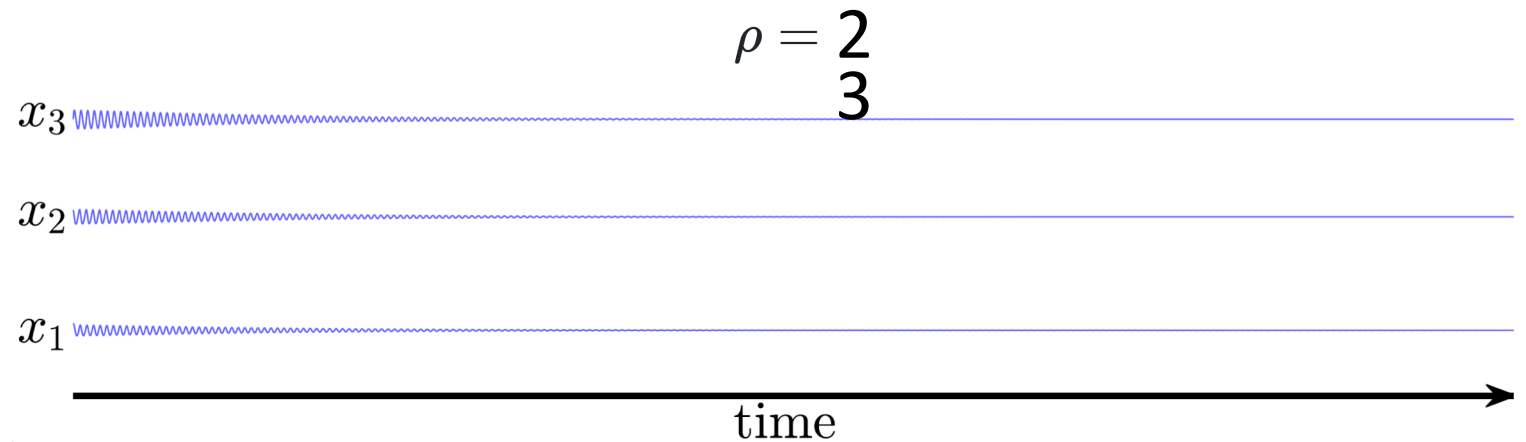
$$\dot{x}_3 = x_1x_2 - \beta x_3,$$

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Lorenz system:

$$\begin{aligned}\dot{x}_1 &= \rho(x_2 - x_1) \\ \dot{x}_2 &= x_1(\rho - x_3) - x_2 \\ \dot{x}_3 &= x_1x_2 - \beta x_3,\end{aligned}$$



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- The Lorenz attractor undergoes a subcritical Hopf bifurcation
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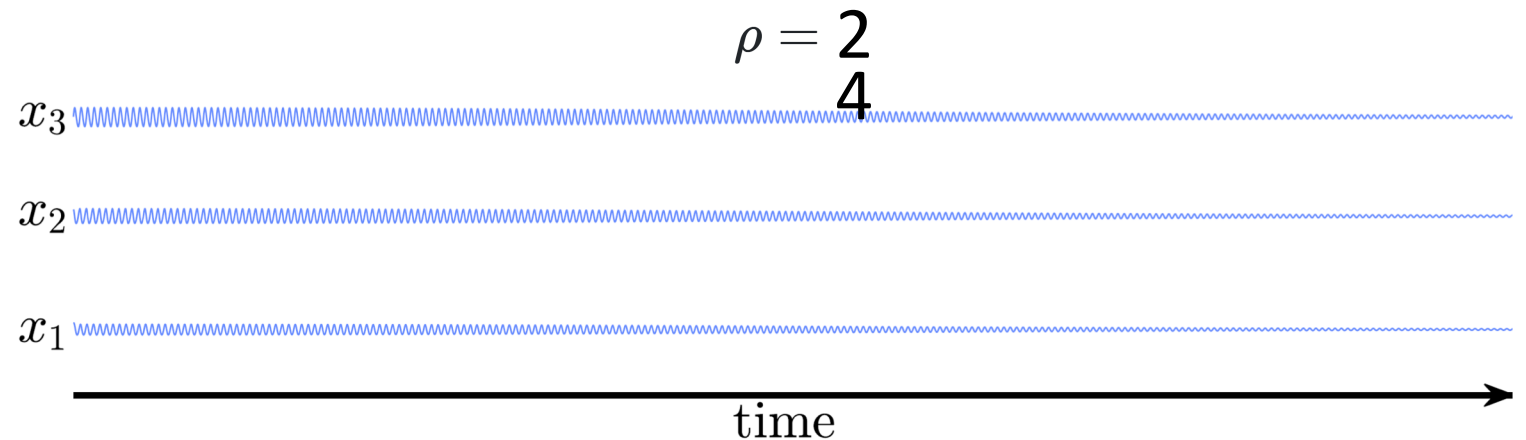
Lorenz

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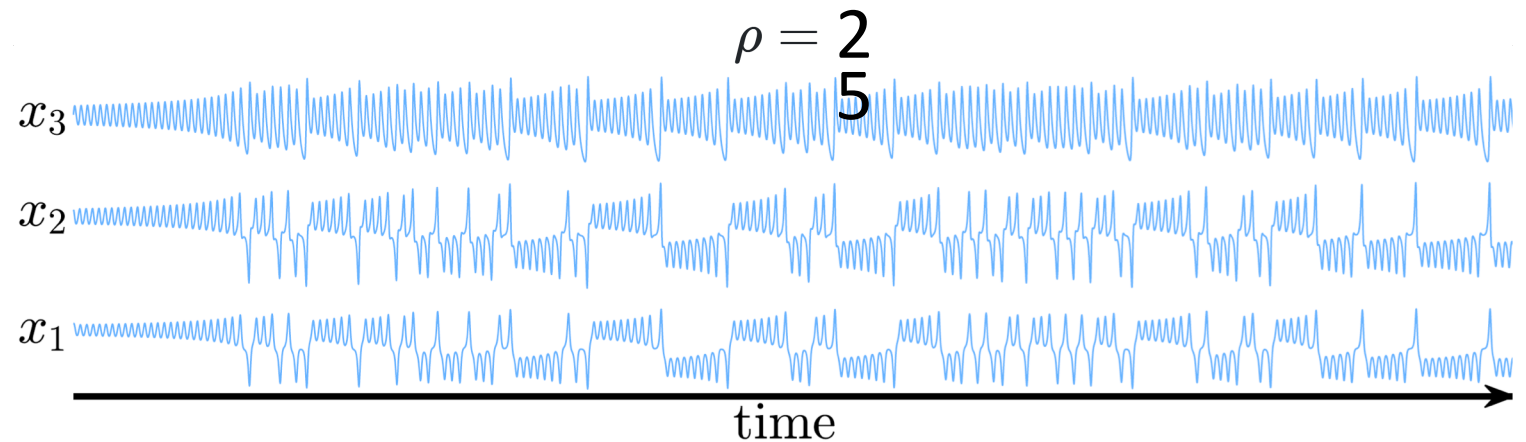
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- How much of a bifurcation can an RNN infer?

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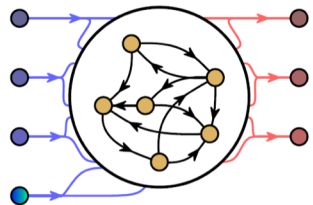
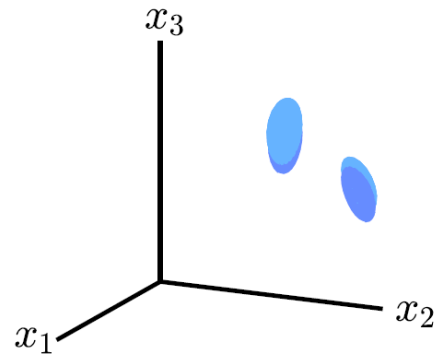
- How much of a bifurcation can an RNN infer?

both fixed points with two stable examples each

training

—  $\rho = 23$

—  $\rho = 24$



# RNNs predict highly nonlinear events by imitating examples

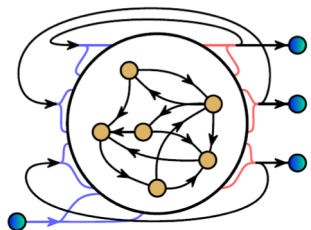
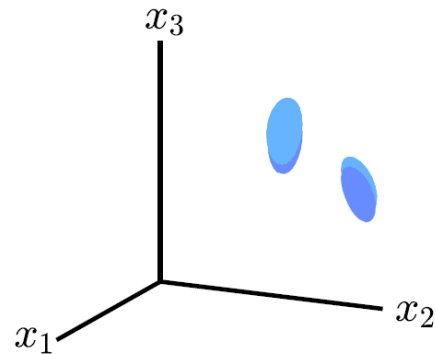
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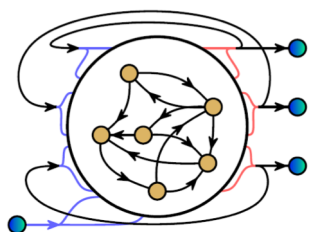
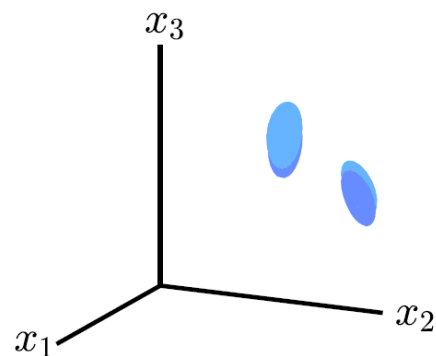
- How much of a bifurcation can an RNN infer?

both fixed points with two stable examples each

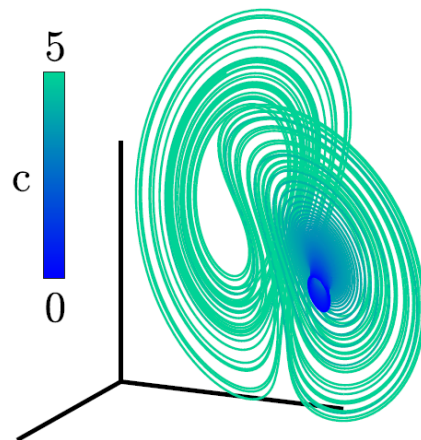
training

—  $\rho = 23$

—  $\rho = 24$



prediction





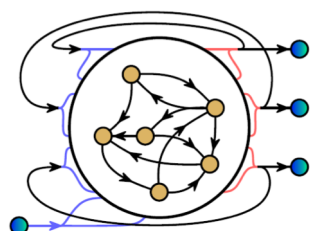
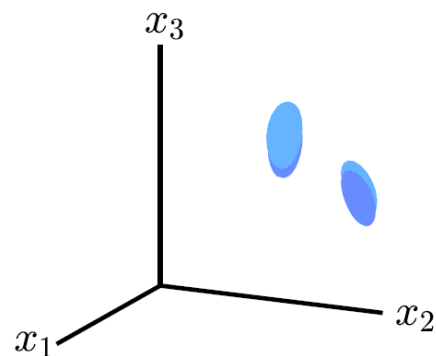
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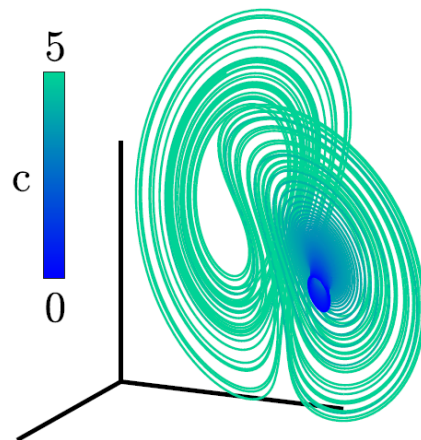
both fixed points with two stable examples each

training

—  $\rho = 23$   
—  $\rho = 24$



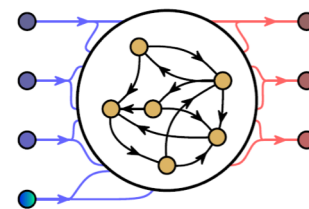
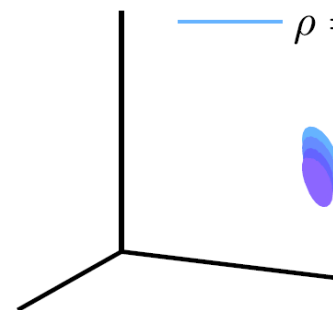
prediction



one fixed point with four stable examples

training

—  $\rho = 21$   
—  $\rho = 22$   
—  $\rho = 23$   
—  $\rho = 24$



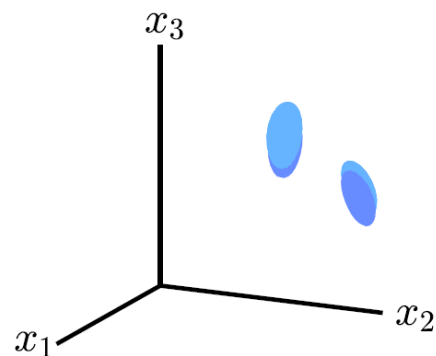
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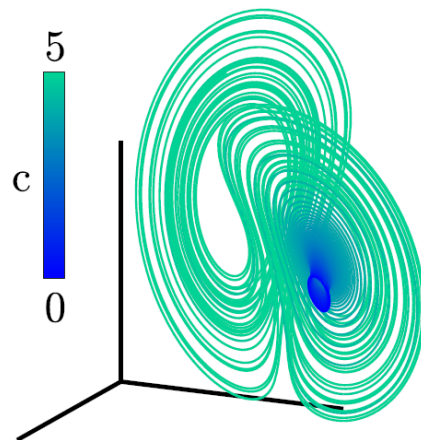
both fixed points with two stable examples each

training

—  $\rho = 23$   
—  $\rho = 24$



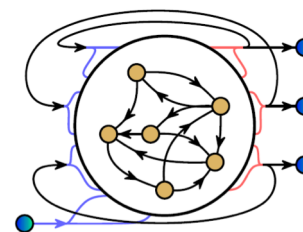
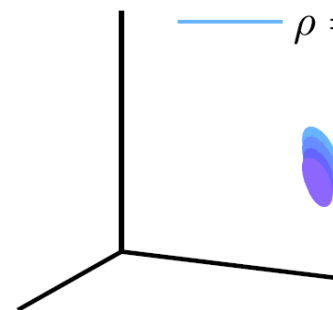
prediction



one fixed point with four stable examples

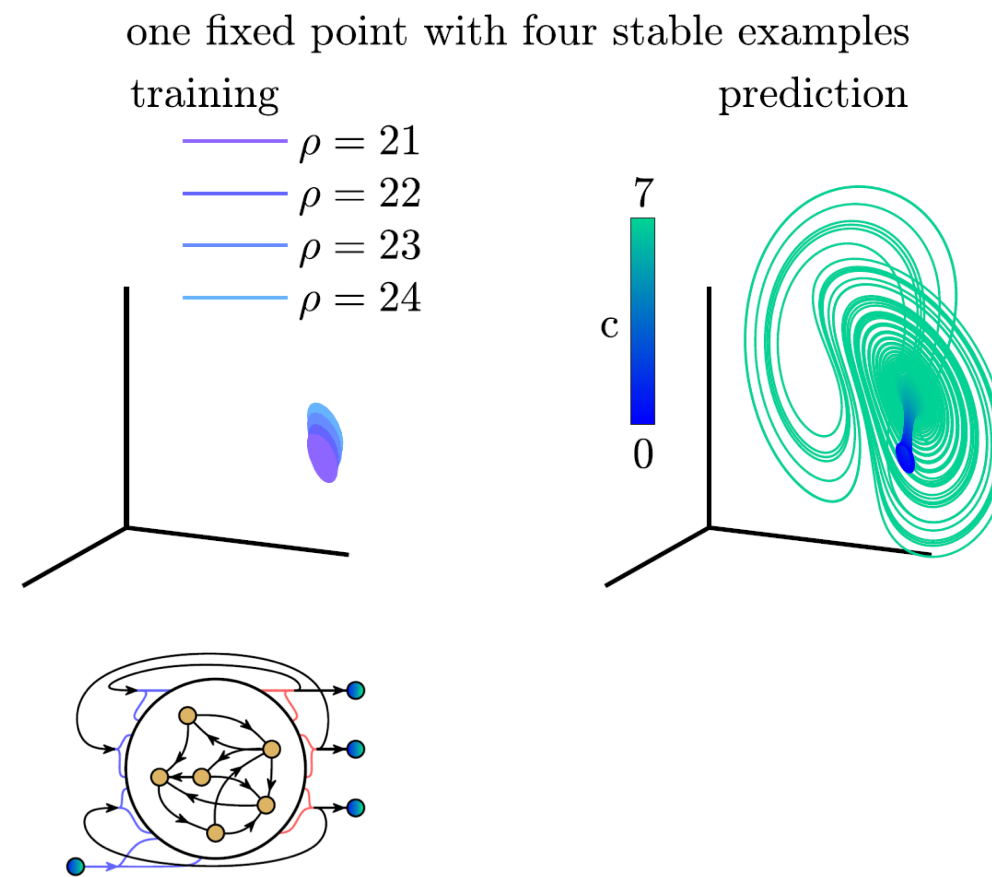
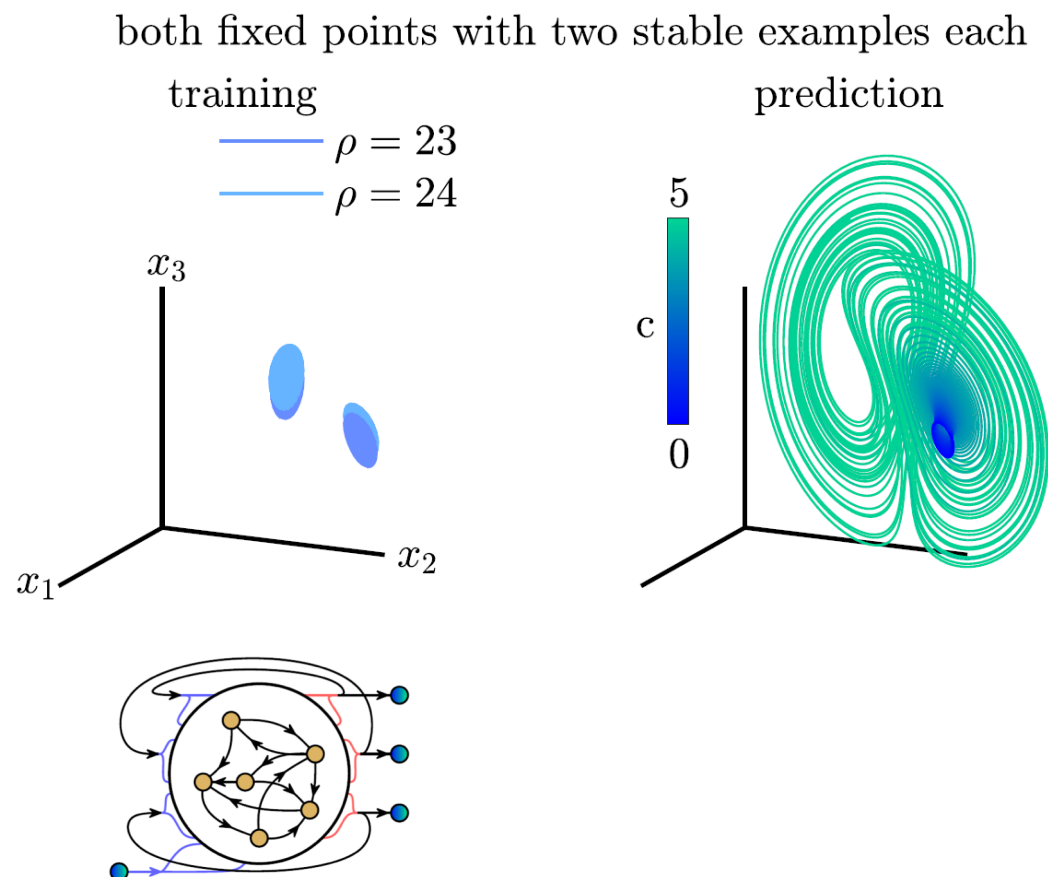
training

—  $\rho = 21$   
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—  $\rho = 24$



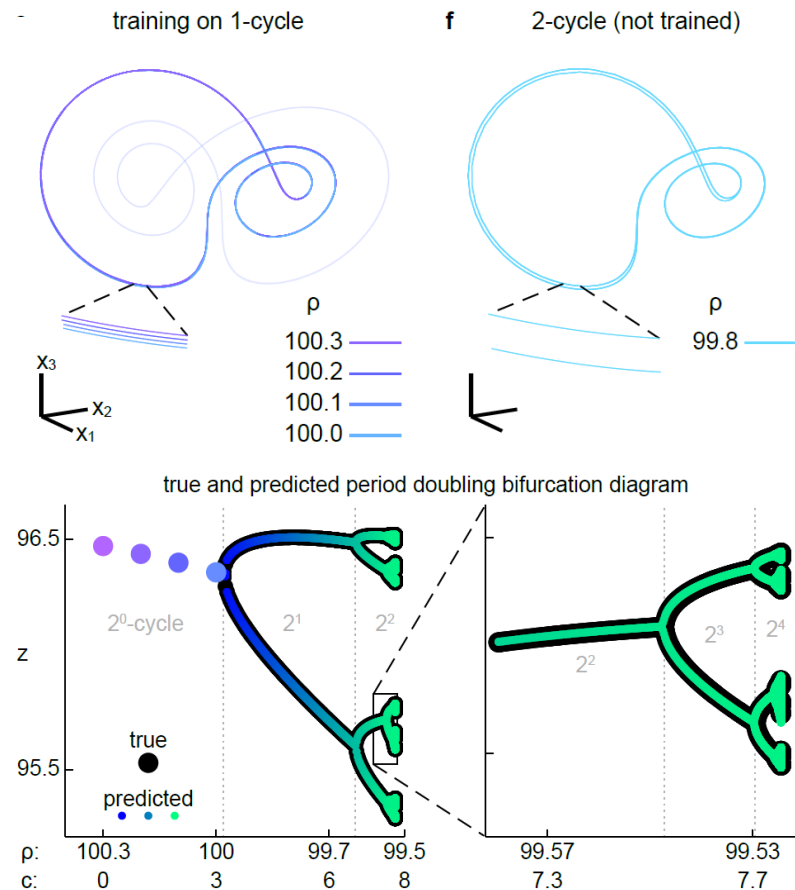
# RNNs predict highly nonlinear events by imitating examples

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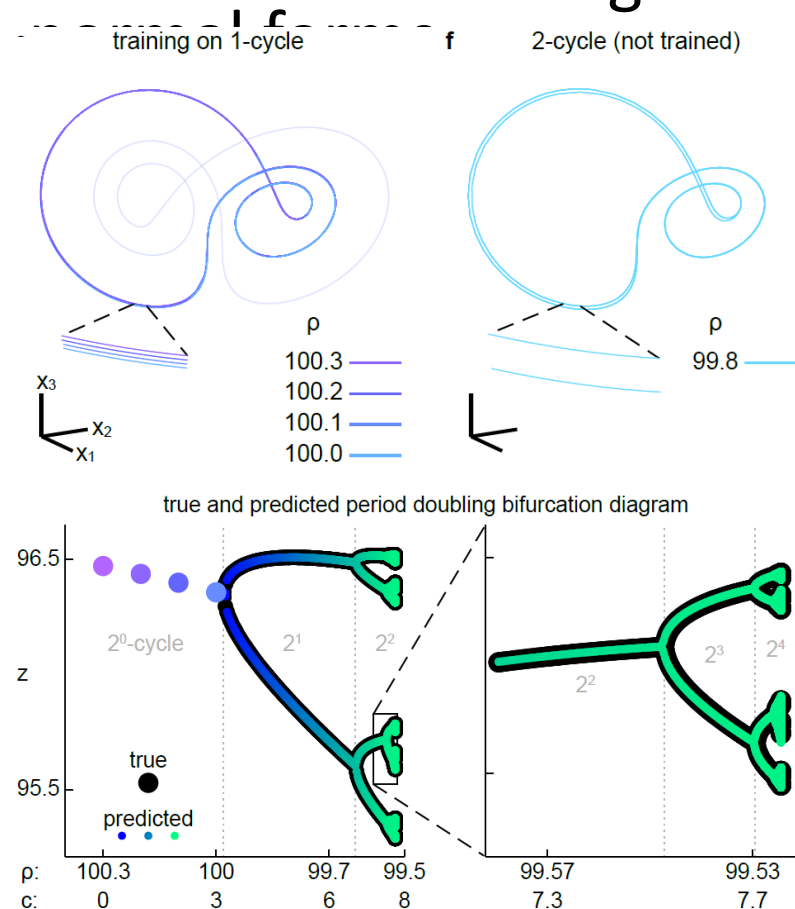
# RNNs predict highly nonlinear events by imitating examples

## Period doubling bifurcation

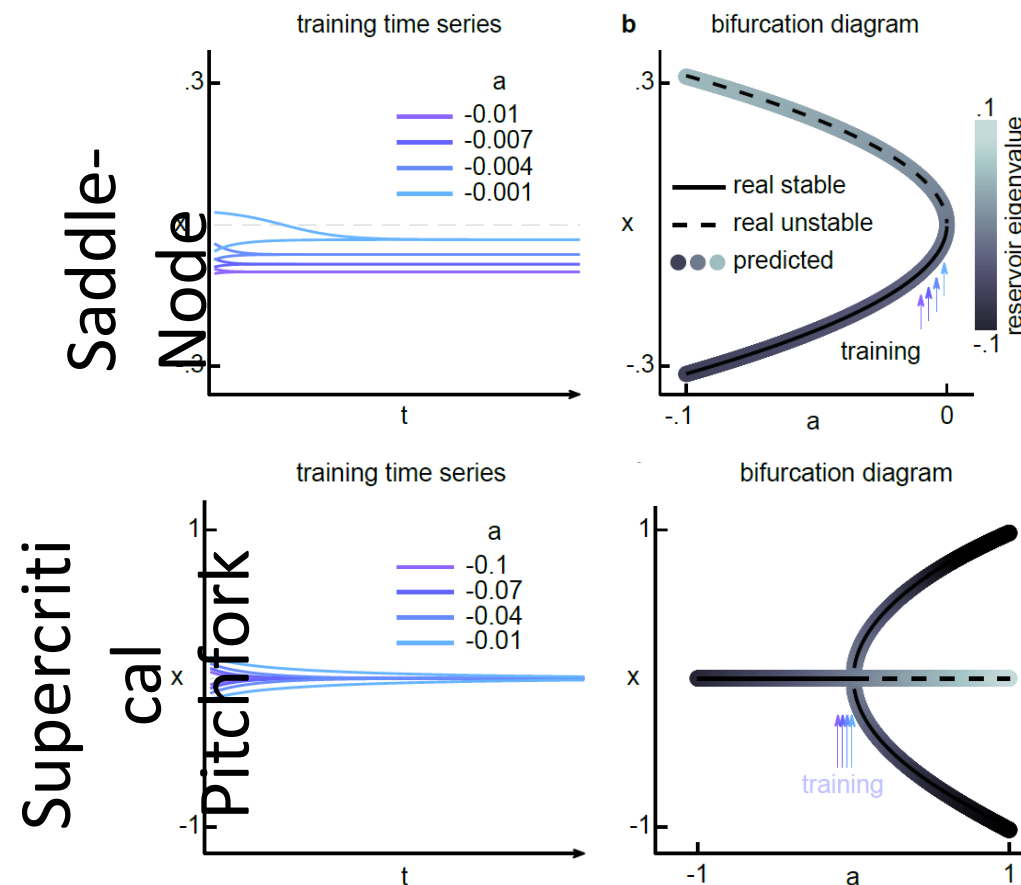


# RNNs predict highly nonlinear events by imitating examples

## Period doubling bifurcation

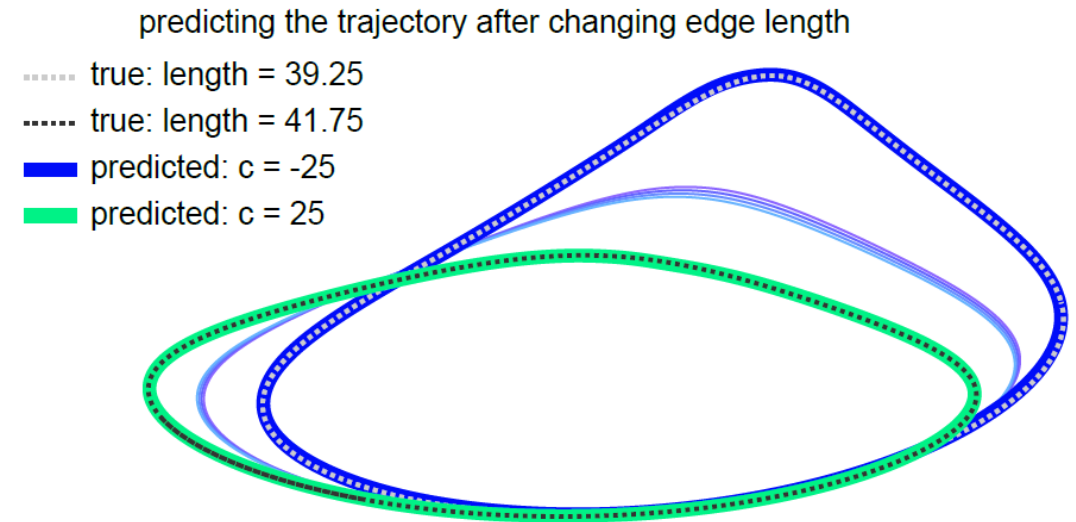
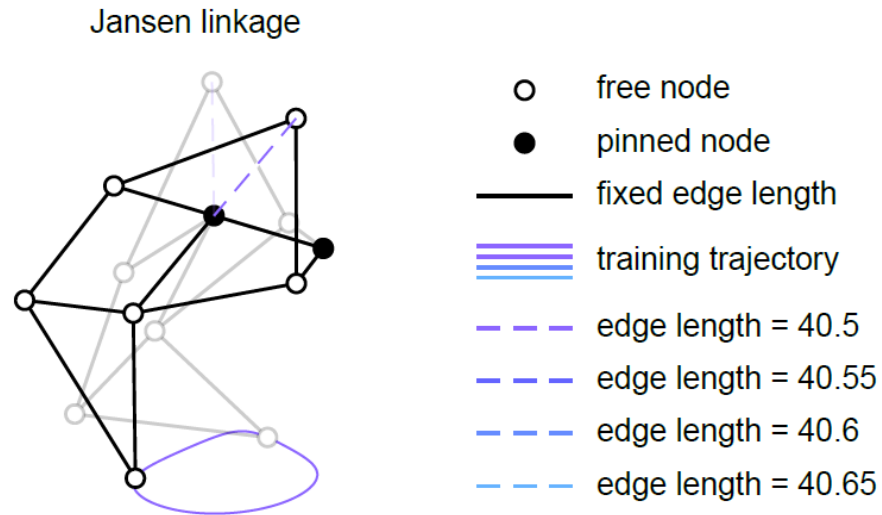


## Bifurcation



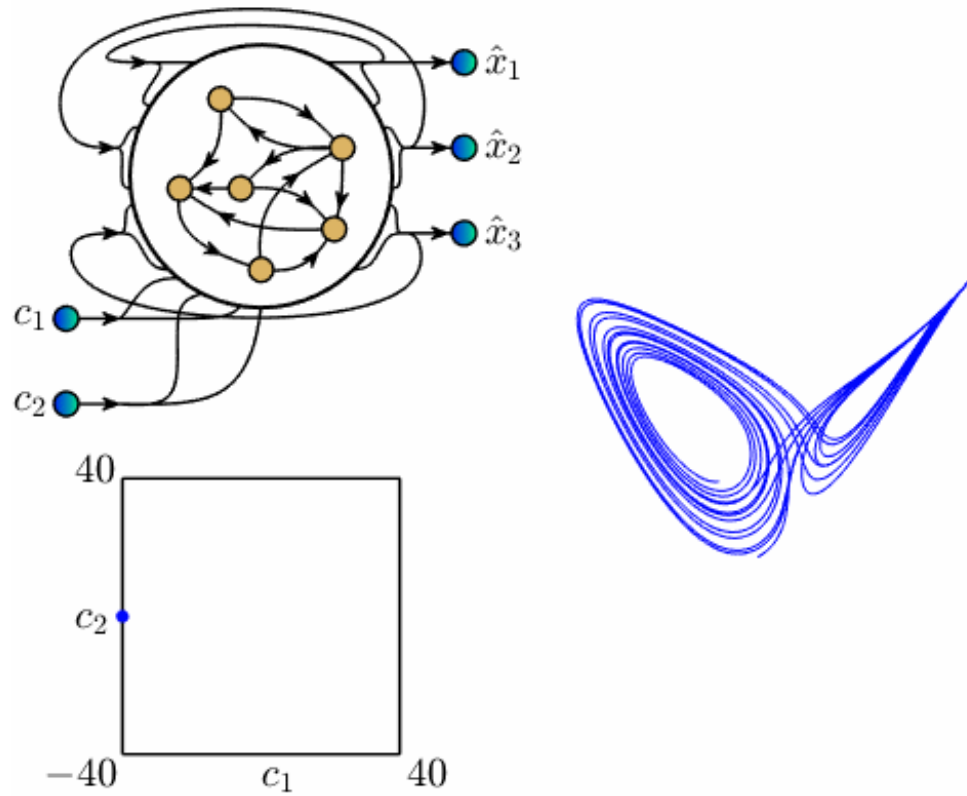
# RNNs predict highly nonlinear events by imitating examples

## Kinematic trajectories



# Flight of the Lorenz

- Translation in  $x_1$  and  $x_3$



# How do RNNs learn translations and transformations?

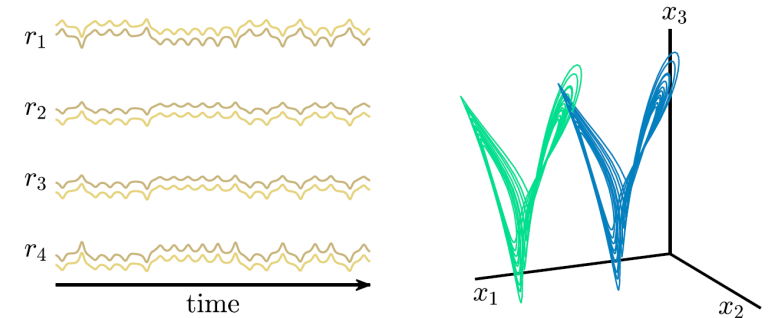
Translation: discrete

$$W\mathbf{r}_c(t) \approx \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} c$$

infinitesimal

$$W d\mathbf{r}(t) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dc$$

$$d\mathbf{r}(t) \approx f(\mathbf{r}(t), W) d\mathbf{c}$$





# How do RNNs learn translations and transformations?

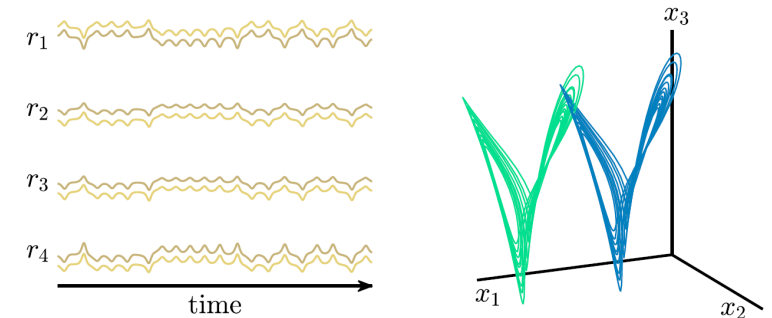
Translation: discrete

$$W\mathbf{r}_c(t) \approx \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} c$$

infinitesimal

$$W d\mathbf{r}(t) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dc$$

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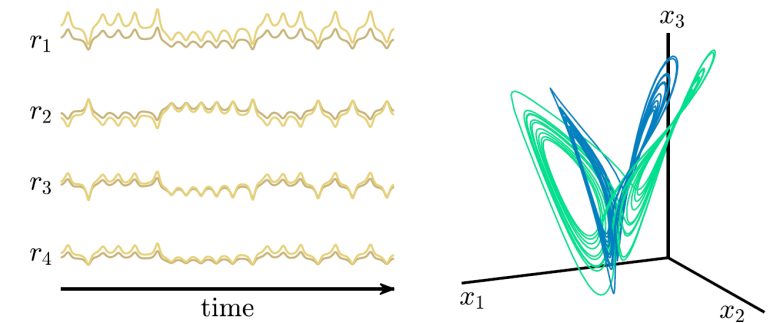
Transformation: discrete

$$W\mathbf{r}_c(t) \approx [I - Tc]\mathbf{x}(t)$$

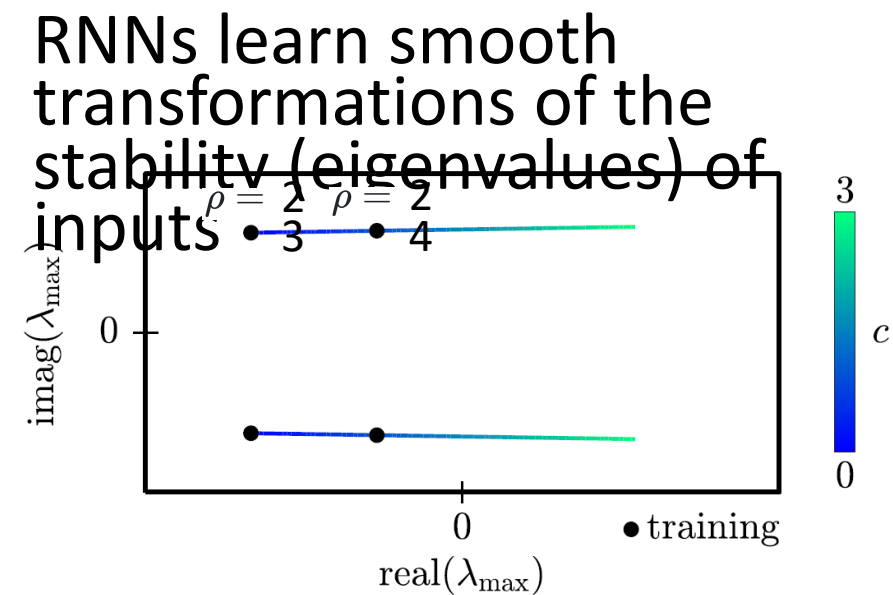
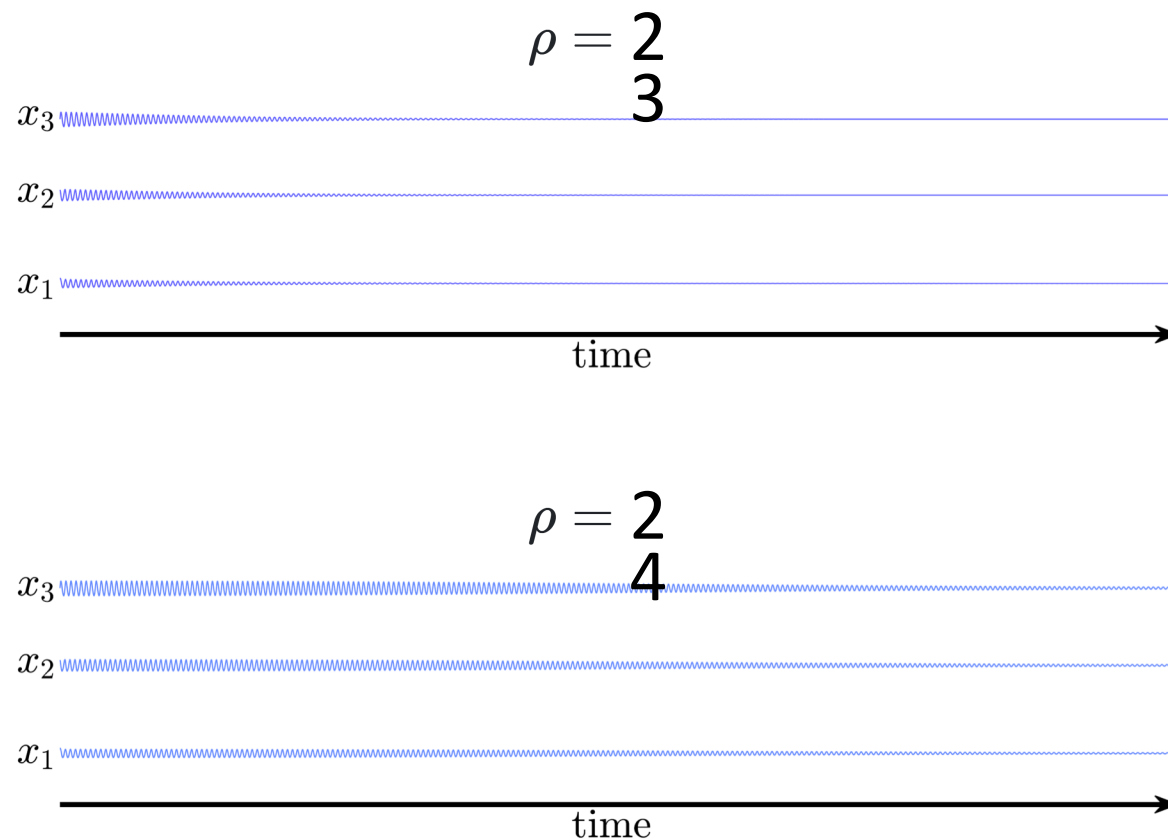
infinitesimal

$$W d\mathbf{r}(t) \approx -T\mathbf{x}(t) d\mathbf{c}$$

$$d\mathbf{r}(t) \approx f(\mathbf{r}(t), W) d\mathbf{c}$$



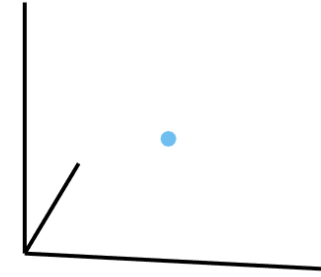
# How do RNNs learn bifurcations?



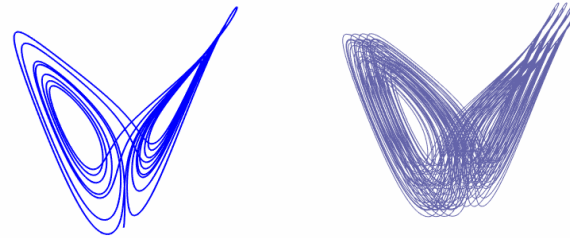
Slower rate of decay  $\Rightarrow$  less negative eigenvalue

# Conclusions: simply by imitating inputs, reservoirs can

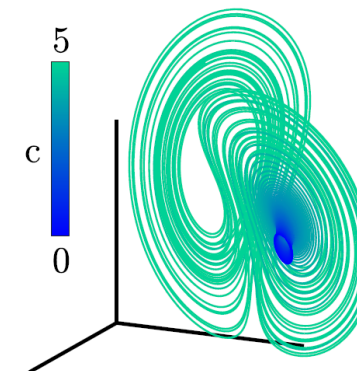
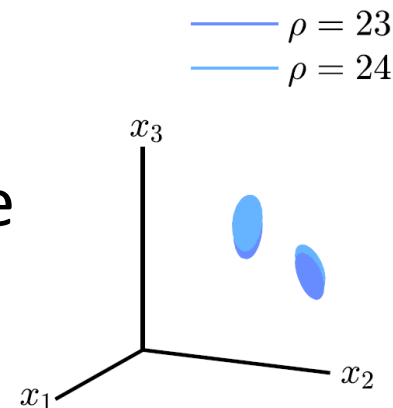
- Sustain complex temporal representations as n



- Translate and transform memo



- Infer global nonlinear structure



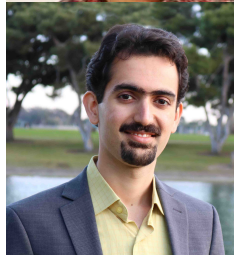
# Thank you!

## Collaborators

Zhixin Lu



Erfan Nozari



George Pappas

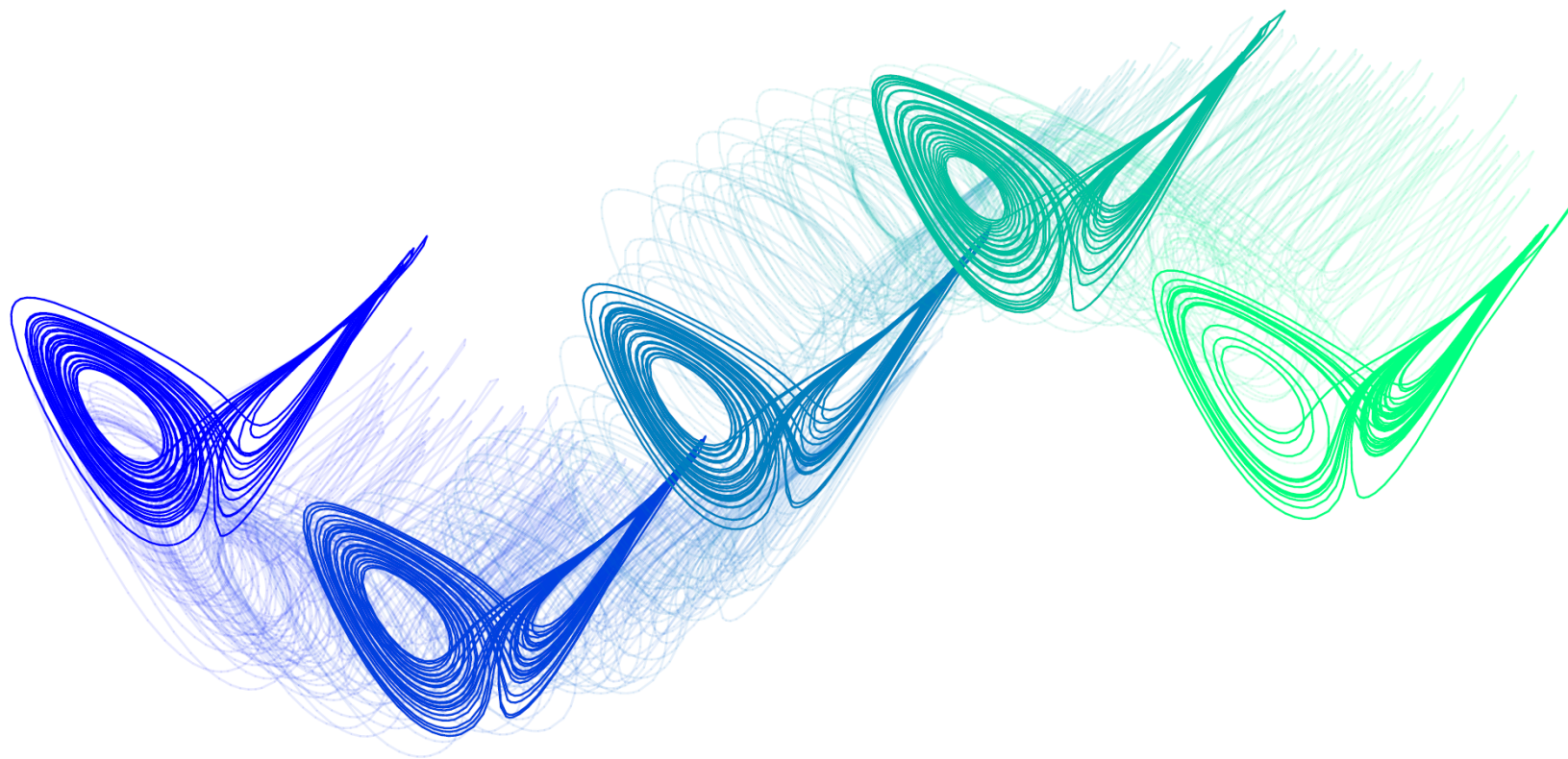


Danielle Bassett



## Funding

NSF GRFP: DGE-1321851



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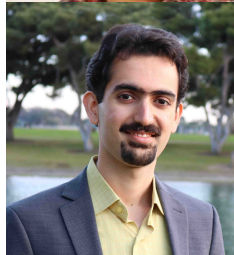
# Thank you!

## Collaborators

Zhixin Lu  
jinsu1@seas.berkeley.edu



Erfan Nozari



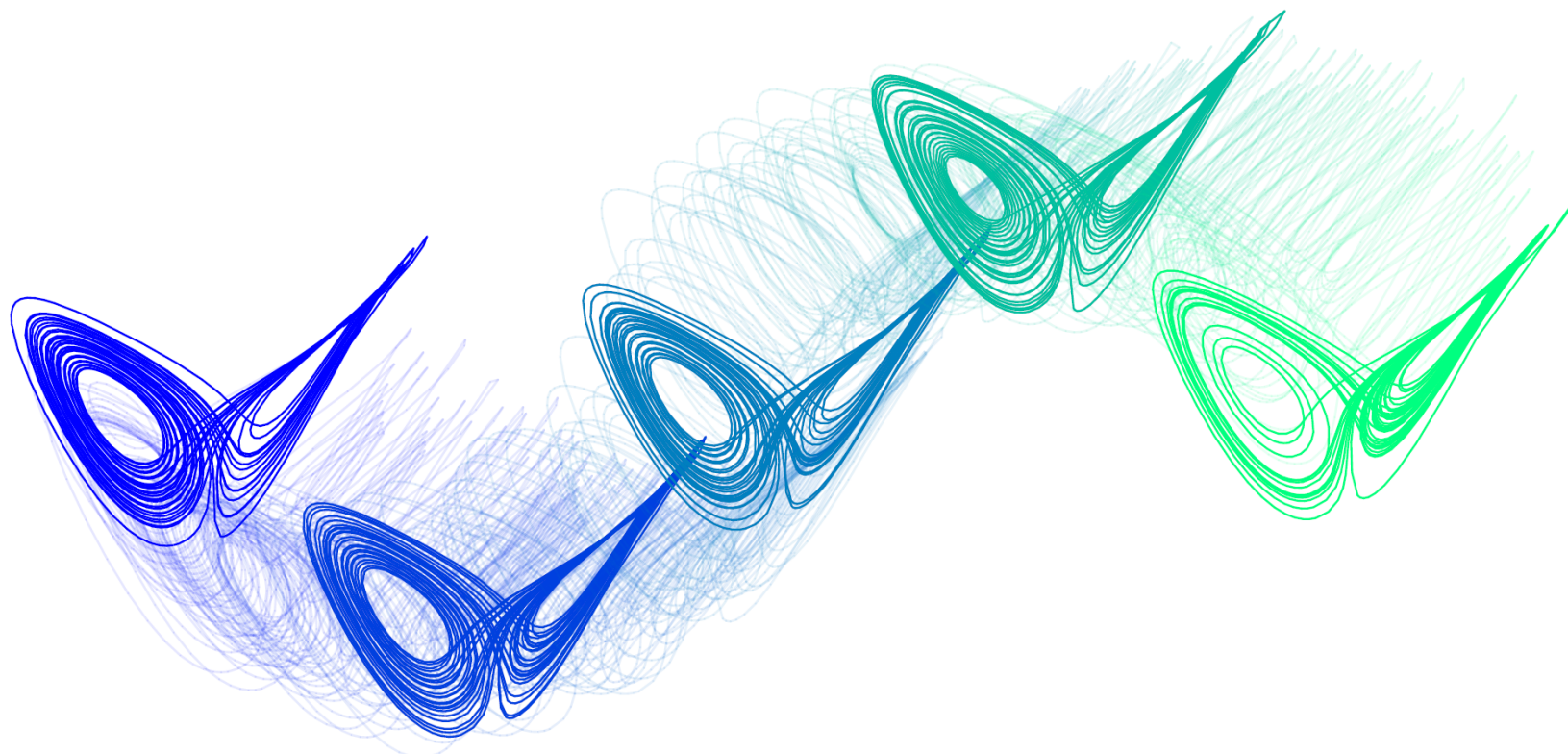
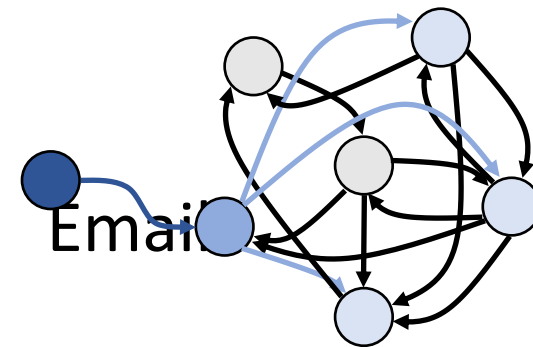
George Pappas



Danielle Bassett



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